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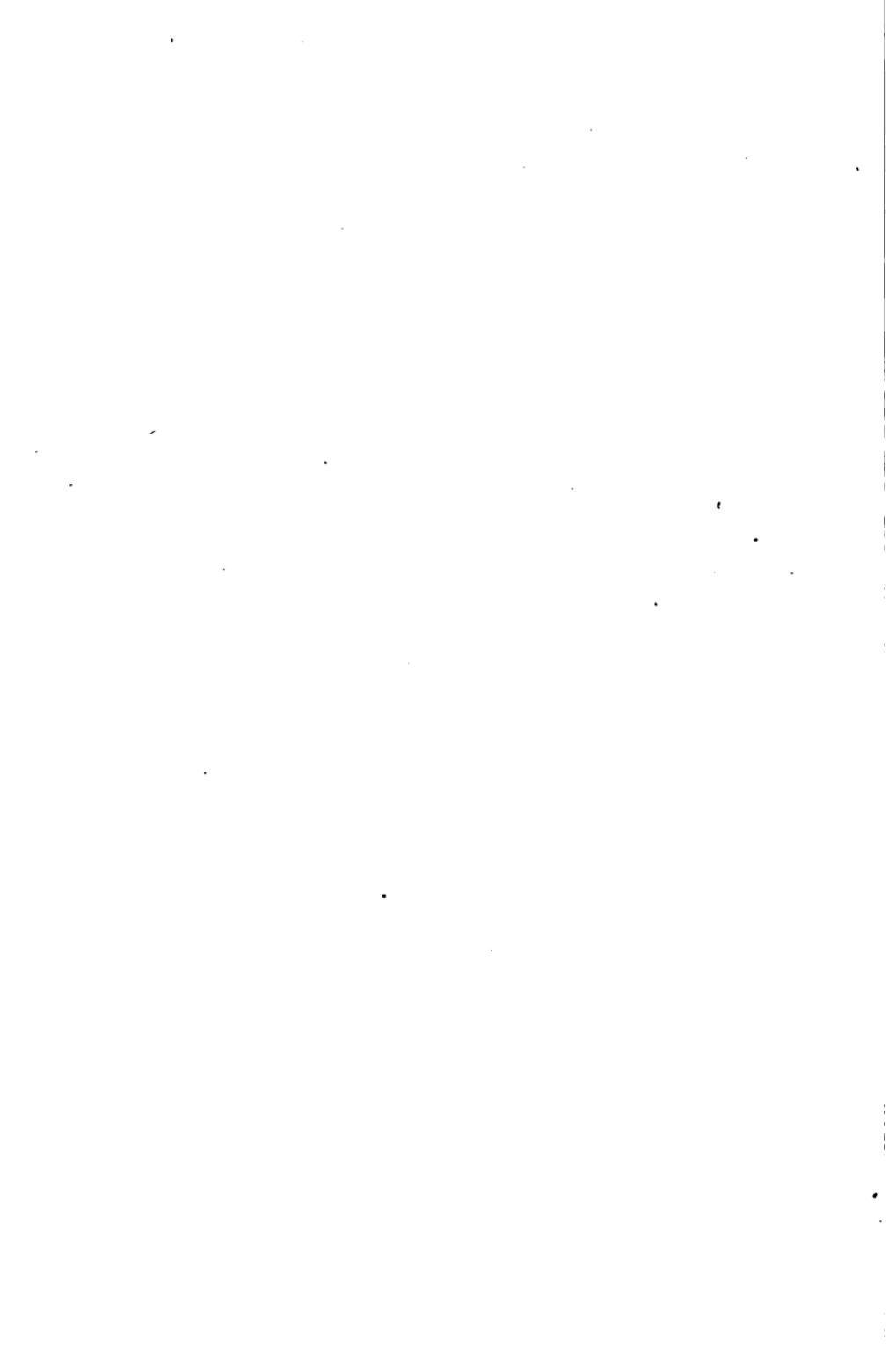
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JUNIOR HIGH SCHOOL MATHEMATICS

BOOK I

BY

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CHARLES SCRIBNER'S SONS

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A

TO THE TEACHER

It is the business of schools to give children during the first six years of school life that kind of instruction in mathematics which will lead them to a quick recognition and a ready knowledge of number combinations and number operations, and then enable them to apply these number combinations and number operations to the solution of simple problems. The instruction for this period, if it is to serve its purpose, must be definite and specific.

At the end of the sixth year the pupil should have a thorough mastery of the number combinations in integers; of the four fundamental operations with integers, common fractions, and decimal fractions; of the common measurements; and of the use of all of these in the solution of problems. Since eternal review is the price of excellence in all mathematical work, especially in computations, this book contains a complete but not lengthy review of the work of the first six years. The reviews are arranged elastically, however, so that the time devoted to them can be determined by the needs of the class. Monotony, the one great drawback of reviews, has been removed by connecting the matter reviewed by historical references of interest, by looking at it from a standpoint of business, by number contests, by using the matter to be reviewed as a background for new work.

Teachers will find the study of mathematics developed here so that every principle is studied at sufficient length to make a lasting impression upon the child and to show at the same time its significant meaning when applied to

the activities of life. The principle is not, however, dropped when it has been thus studied but is brought continually into play in the problems following. A careful scrutiny of the exercise material will show clearly this feature of the book. Much spiralization has been found to lead to loose, disconnected ideas of the whole subject, one of the things for which mathematics and mathematics teaching is now very justly criticised. On the other hand, it is equally bad to hermetically seal each topic in a separate compartment and place this on the shelf to gather dust after it "has been completed." But extremes have been avoided.

During the transition period of the Junior High School the pupil should be led to rationalize and to generalize. He should now be asked to discover and observe for himself the interrelationships found in his school activities and in the life about him. His mathematics from this point on should train him to study every question that arises from its quantitative side. He should be led to solve the various interrelated problems involving quantity as he sees them around him. Thrift and economy and social relationships are especially stressed in the problems and under suitable topics. Such a treatment lends itself to a continuous and progressive appeal to the child to know and to get the qualities that make for good citizenship. Therefore the author treats **mathematics** throughout as an **instrument for use**, and to that end problems and projects to which each mathematical principle is applied have been graded to conform to the successive stages of development of the child mind and of child activity.

The mode of instruction is less and less specific and more and more suggestive. To accomplish this, simple language has been used, and the subject expanded by easy stages, so that the pupil may be given self-reliance and at the same

time constant encouragement to be ever on the alert for new discoveries. His senses are employed to introduce him to rationalization and to generalization according to the best-established psychology. Geometrical construction work accordingly receives special attention in this the first book of the series. Literal numbers are studied for their applied value in stating geometrical formulas. The pupil meets here for the first time a new form of mathematical expression, a shorthand symbolical language.

By way of emphasizing some of the more practical features of the book, the author begs to call attention to the following facts: In connection with computations, checks are given a prominent place. Estimates and approximations as well as other forms of simplifying number work are used continually. The three forms of fractions—common fractions, decimal fractions, and percentage—while placed in separate chapters for convenience, are still treated as fractions; merely different forms of expression. The relations between the three forms are clearly brought out. Weights and measures are placed in the book only for those needing review in that work. The metric system is taken up to show its simplicity and the close approximation of the metric units to our English units. Literal numbers are confined to monomials, which are used principally in stating laws and geometric formulas. A new generalization arises in the extension of the number system to negatives. In the geometric work the appeal is made particularly to the constructive and discovering nature of the child. The work requires much drawing and presents many opportunities for sense judgment and will continually raise questions for investigation.

The author desires here to acknowledge the very valuable aid rendered by Mr. W. H. Keller, of the Kansas State

Normal, and Miss Lena B. Hansen, of the Enid, Oklahoma, High School, who criticised the entire manuscript; by Mr. E. A. Lewis and Mr. Howard Patterson, who assisted in preparing the material for the cuts; as well as by the large body of students and teachers who have given both inspiration and suggestions.

THEODORE LINDQUIST.

EMPORIA, KAN.,

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JUNIOR HIGH SCHOOL MATHEMATICS

BOOK I

I

WHOLE NUMBERS



1. Early Counting.—Since the earliest times men have counted and measured. They have asked the questions, "How many?" and "How much?" Even the savage counted, "one, two, three, four, heap many." In the earliest times counting was carried on by the use of pebbles, sticks, and fingers. Three people were needed to count upon their fingers beyond one hundred, as is shown in the picture. The first counted the units, the second the tens, and the third the hundreds. Our present system of counting in units, tens (tens of units), hundreds (tens of tens), thousands (tens of hundreds), and so on, has likely come from this early finger counting by tens. The nine numerals, 1, 2, 3, etc., are often called **digits**, a name we also use for fingers.

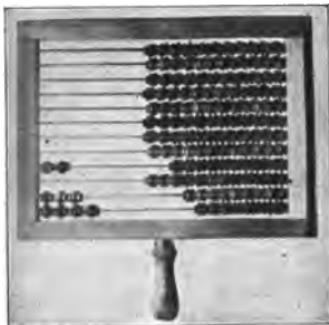
2. Hindu-Arabic Number System.—Notches in sticks, knots in ropes, and other crude means were used in the early times to record numbers. Marks were also made upon stones and in the sand. The marks were not combined to form numbers as we do now, but a separate one had to be used for each number. The following are a few that were used:

						
1	2	3	5	10	30	40
						
60	80	100	160	200	1050	

During the early centuries of the Christian era the Hindus, the people of India, thought out the idea of using only a few marks and combining these so as to form different numbers. The nine numerals they used have changed very much, finally becoming what we use to-day, but the Hindus used their numerals in the same way as we do ours. To write four hundred forty-four we use only one figure, 4, but the 4 takes different values in the three positions in 444. With 3, 5, and 7, six different numbers can be formed: 357, 375, 537, 573, 735, and 753. What numbers can be formed from 4 and 6? from 2, 5, and 8?

For about three or four hundred years the Hindus used the nine numerals to form numbers before they invented the zero and its use in forming numbers. They could write 345 but not 340 nor 305. Numbers could be expressed upon an abacus without the zero. On the next page is an abacus such as you may have used as a child or may have seen in use by the Chinese. This one records 2,034. How would you record 1,250? 634? 3,092? They also used small

stone tablets strewn with sand, which were marked off into columns for recording numbers. For hundreds of years all computations had to be carried out upon an abacus or a stone tablet.



In trading with the Hindus, the Arabs learned these simple new numerals and their use. The Arabs also traded with the peoples of southern Europe, to whom they brought the new numerals. In this way the Hindu numerals were spread over the entire civilized world. Because the Arabs brought them into Europe, the numerals have been incorrectly called **Arabic**.

We use the nine numerals to form numbers in much the same way as we use letters to form words. The nine numerals and the zero are thus a sort of **number alphabet**. The same numerals are used in the different countries, which is not the case with the letters. The printed and written numerals are very nearly alike, while the printed and written letters differ very much.

About two hundred fifty years ago the words **million**, **billion**, **trillion**, **quadrillion**, and so on, came into use in northern Europe. In the United States large numbers are separated into periods of three, as is here shown:

365, 972, 105, 635, 307.
trillion—billion—million—thousand—units.

3. Roman Number System.—In the Roman number system I, V, X, L, C, D, M stand for 1, 5, 10, 50, 100, 500, 1,000. A dash placed above a symbol multiplies it by 1,000. Thus, $\bar{V} = 5,000$. A symbol placed before a larger one is subtracted from it. Thus 4 is made by IV, except some clocks have IIII; 9 by IX; 40 by XL; 90 by XC; 400 by CD; 900 by CM. In all other cases a symbol is placed after an equal or larger one and added to it. Thus, 51 is LI; 56 is LVI; 65 is LXV; 70 is LXX.

The Roman number system was invented by the men of ancient Rome long before the Hindu system, of which we have just read. The Roman number system was the chief number system until 1200 A. D. Now it is used only where different forms of numerals are needed. Where have you seen the Roman numerals?

EXERCISES

Separate the following numbers into periods and read them:

- | | | |
|--------------|----------------|----------------|
| 1. 457328301 | 4. 7235601482 | 7. 1036525973 |
| 2. 61732870 | 5. 15234090451 | 8. 4307349518 |
| 3. 230681934 | 6. 3066073215 | 9. 30460510230 |

Read the following and express each in Hindu numerals:

- | | | |
|--------|---------|------------|
| 1. XLV | 3. CXLI | 5. CMXIV |
| 2. LIX | 4. CLIV | 6. MMCXVII |

Write the following in Roman numerals:

- | | | |
|--------|---------|---------|
| 1. 65 | 3. 509 | 5. 1255 |
| 2. 135 | 4. 1560 | 6. 1919 |

Can you make boys' and girls' names out of the following:

- | | | |
|---------------|--------------|-----------------|
| 1. 1000ARY | 3. 500I100K | 5. WI5050IA1000 |
| 2. E10001000A | 4. 50I5050IE | 6. 500A5I500 |

4. Reading and Writing Numbers.—Large numbers usually arise only in connection with national affairs, as stating a country's area in square miles, a nation's debts, and so on. Ordinary numbers contain six digits or less and are read differently by business and scientific men from what you have been used to in reading numbers. For instance, 456291 is read "forty-five, sixty-two, ninety-one"; 73024 is read "seven, thirty, twenty-four." Notice that "and" is never used; that comes only in decimals, which we shall study later.

Telephone numbers are called to the operator in yet a different way. Each digit is named separately so that 3648 is called "three, six, four, eight."

It is often necessary to write out numbers in words, as on bank checks. For instance, 356 is written three hundred fifty-six, while 3064 is written three thousand sixty-four. Again notice that and has not been used. Such numbers as fifty-six and forty-one are compound words and must be written with the hyphen.

EXERCISES

Read the following numbers as a business man would:

- | | | | |
|---------|------------|------------|-------------|
| 1. 5632 | 7. 456139 | 13. 408671 | 19. 450634 |
| 2. 3741 | 8. 167013 | 14. 480631 | 20. 14603 |
| 3. 1056 | 9. 67903 | 15. 140036 | 21. 1679045 |
| 4. 5603 | 10. 560731 | 16. 56701 | 22. 470631 |
| 5. 2405 | 11. 405713 | 17. 378901 | 23. 370825 |
| 6. 3016 | 12. 478305 | 18. 468329 | 24. 468329 |

Read the first six numbers above as telephone numbers.

Write the first six numbers above in words.

Let each pupil find some large numbers in papers or magazines to bring to school.

5. Addition Combinations.—Accuracy and speed in addition require that the forty-five addition combinations, as $4 + 7$, be memorized thoroughly. You must know that $4 + 7$ and $7 + 4$ equal 11 as quickly as you do that cat. Do not memorize the combinations as tables of 2's, 3's, etc., but learn each combination alone as a separate fact. That is the only way you use the combinations.

EXERCISES

Give the sum of each digit in the following with the one just below it. Give the sum of each digit with the one next to its right. Find the sum of each column; add both upward and downward to check your result.

5	7	8	9	6	7	5	9	7
8	6	9	4	3	4	8	8	9
9	7	8	6	9	7	5	9	4
7	6	5	8	6	8	7	5	8
6	8	9	4	7	6	5	8	9
4	9	6	7	8	5	9	7	6
3	8	4	5	7	9	6	8	5
7	5	6	8	9	6	8	4	9
8	7	5	6	8	9	7	9	4
9	6	9	3	8	5	6	9	5
4	7	4	8	5	8	7	6	7
7	4	9	4	9	5	8	3	8

6. Adding 9's and 8's.—In adding 9 to a two-digit number note that the ten's digit is merely increased 1, while the unit's digit is decreased 1. In adding 8 the ten's digit is increased 1 while the unit's digit is decreased 2.

EXERCISES

Without the use of pencil and paper give as rapidly as possible the sums of the following:

1.	53	46	60	28	36	46	29
	9	8	9	8	9	8	9
	<hr/>						
2.	48	39	78	69	75	56	37
	9	5	4	6	8	9	9
	<hr/>						
3.	76	59	43	67	57	29	18
	8	4	8	9	9	6	9
	<hr/>						
4.	41	38	95	81	24	38	23
	8	4	8	9	9	8	9
	<hr/>						
5.	68	73	64	29	36	28	37
	5	9	9	5	8	5	8
	<hr/>						
6.	49	78	58	29	39	59	69
	6	7	9	8	7	8	7
	<hr/>						
7.	56	89	74	38	27	64	75
	8	5	9	7	9	8	9
	<hr/>						

7. **Addition of Columns of Numbers.**—First find the sum of the units' column to be 24, or 4 units and 2 tens. The 4 is written under units' column and the 2 is written above the tens' column. The tens now add up to 31, or 1 ten and 3 hundreds. The 1 is written under tens' column, and the 3 above the hundreds' column. Further additions are carried on in the same manner. Check all addition by adding both upward and downward.
- | | |
|-------|------|
| 232 | |
| 6573 | |
| 5759 | |
| 7497 | |
| 6885 | |
| 26714 | sum. |

8. A Number Game.—The teacher appoints 3, 4, or 5 captains so as to divide the class into nearly equal teams. The captains choose sides. When pupils are ready with pencils and paper the teacher reads from 5 to 20 exercises.

When a pupil has completed and checked his work he raises his hand. The first receives a score of 100, the next 99, and so on. Those who do not finish within the time decided upon by the teacher receive no score. The teams exchange papers. The teacher then reads the correct results, 20 being taken from each score for every mistake, as long as any score remains. Each captain brings to the teacher the corrected papers of his team. From these papers the teacher reads the scores of one team, which all the pupils write down and add. This is divided by the number on the team to get the average. The average score of each of the other teams is found in the same manner. The team with the highest average score wins.

New teams may be selected for each number game or the same teams may compete for a month or more and the record of the teams be averaged after each game.

The pupils of one room called their teams the Reds, the Browns, and the Whites. Select names for your teams.

For the first game use exercises in writing numbers and in adding columns of numbers.

$$\begin{array}{r}
 4\ 5\ 4 \\
 5\ 6\ -3\ 4 \\
 2\ -9\ -7\ -8 \\
 4\ 7\ 9\ -3 \\
 3\ 4\ -8\ -7 \\
 7\ -2\ 4\ 6 \\
 2\ 5\ 8\ -9 \\
 4\ -7\ -8\ -6 \\
 \hline
 3\ 1\ 5\ 1\ 3
 \end{array}$$

9. Rapid Addition.—Begin at the top of units' column, $4 + 8 = 12$. Drop the ten and place a dash after the 8 to indicate this. Add the remaining 2 to 3 and this to 7, giving 12. Place a dash after the 7 to indicate that 10 has been dropped out of 12. As before, $2 + 6 + 9 = 17$. Place a dash after 9. Again $7 + 6 = 13$. Place a dash after 6 and write the unit digit, 3, under the line. Looking up units' column we see 4 dashes, showing that 4 is to be carried to tens' column. The 4 is written above tens' column. Tens' column and the other columns are added just as units' column.

Many who do rapid work in addition look for sets of digits in a column whose sum is 10, as, $6 + 4$ or $2 + 3 + 5$. Some also look for repetition of digits, as four 7's equal 28 or three 9's equal 27.

Read the following numbers as a business or scientific man, copy each exercise, find the sum, and check:

EXERCISES

1. 297357	2. 606745	3. 923893	4. 962375
431473	248338	586729	758143
675739	564895	486729	524387
899273	695790	764359	853694
248352	737856	834901	268407
759675	488406	894567	964583
686584	363358	597368	408698
459628	423672	379105	744960
744756	758948	795684	753496
865478	895469	694373	795385
<u>284325</u>	<u>389217</u>	<u>564823</u>	<u>2506719</u>
5. 957834	6. 853766	7. 853465	8. 964573
863572	865683	276285	533859
598256	754983	524267	649365
876379	364573	746843	955345
497683	174975	583677	736894
593584	597359	856482	485749
698456	292247	497356	369468
852847	479358	528467	535318
427236	497495	934873	916239
369457	596349	489565	157238
<u>470307</u>	<u>934086</u>	<u>470785</u>	<u>670890</u>

10. Business Method of Subtraction.—When we are learning that $4 + 7 = 11$ we should also be learning that 4 must be added to 7 to make 11, and that 7 must be added to 4 to make 11. In finding how much must be added to 4 to make 11, we say that we **subtract 4 from 11**. Subtraction is the reverse of addition.

Suppose that you have made a purchase for 55 cents and have given the clerk a dollar in payment. In giving you

the change due you the clerk will say, "55 cents and 20 cents (handing you two dimes) make 75 cents, and 25 cents (handing you a quarter) make a dollar." The clerk adds enough money to the amount of your purchase

to equal the money you gave to him. What is the advantage of making change in this way?



EXERCISES

1. How much must be added to 8 to make 15? to 9 to make 13? to 7 to make 12? to 8 to make 17? to 5 to make 11?

2. Supply the missing numbers in the following subtractions:

$$\begin{array}{r}
 35 & 54 & ?? & 32 & ?? & 46 \\
 ?? & ?? & 35 & ?? & 27 & ?? \\
 \hline
 13 & 32 & 23 & 10 & 52 & 14
 \end{array}$$

3. How will I receive change from a clerk to whom I have given a dollar if my purchase is 35¢? 45¢?

4. How will a clerk give me change from a five-dollar bill if my purchase is \$2.35? \$2.85? \$3.15? \$3.65?

11. Form of Subtraction.

Minuend 745 3 added to 2 makes 5; write 2 under 3. 6 added to 8 makes 14; write 8 under 6. 1 (carried from 14) added to 2 gives 3, to which 4 is added to make 7; write 4 under 2.
Subtrahend 263
Difference 482

EXERCISES

Make the following subtractions by the above method:

1. 547	3. 7658	5. 7438	7. 4563	9. 6473
<u>351</u>	<u>4358</u>	<u>3215</u>	<u>2745</u>	<u>2594</u>
2. 647	4. 7456	6. 5374	8. 6372	10. 5436
<u>324</u>	<u>5246</u>	<u>2457</u>	<u>2485</u>	<u>1619</u>

11. If you were a clerk in a store, how would you give change for the following purchases:

One dollar given to pay for a purchase of 65 cents.

Five dollars given to pay for \$3.25.

Five dollars given to pay for \$1.45.

Ten dollars given to pay for \$6.35.

12. **Excess of 9's of a Number.**—If a number is divided by 9 and the sum of its digits is divided by 9, the remainders will be the same. $756,834 \div 9$ leaves 6 as a remainder; $7 + 5 + 6 + 8 + 3 + 4 = 33$, and $33 \div 9$ leaves 6 as a remainder. This remainder is called the **excess of 9's** of 756,834. Finding the excess of a number is also called "casting out the 9's." Which of these two ways of finding excesses is the simpler?

Find the excesses in each of the numbers in the above exercises 1 through 10.

13. Checking Subtraction.—First find the excess of the minuend. From this take the excess of the subtrahend. The number thus found equals the excess of the difference, if the subtraction is correct.

- 745 excess 7; $7 + 4 + 5 = 16$ and $16 \div 9$ gives remainder 7.
263 excess 2; $2 + 6 + 3 = 11$ and $11 \div 9$ gives remainder 2.
482 excess 5; $4 + 8 + 2 = 14$ and $14 \div 9$ gives remainder 5.
 $7 - 2 = 5$. What is your conclusion?

If the remainder had been found to be 284, its excess would still have been 5 and the error in subtraction would not be seen. Such errors are very rare and you may still depend upon this simple check.

In case the excess of the minuend is less than the excess of the subtrahend, we must add 9 to the excess of the minuend before subtracting excesses.

- 875 excess 2; $8 + 7 + 5 = 20$ and $20 \div 9$ gives remainder 2.
564 excess 6; $5 + 6 + 4 = 15$ and $15 \div 9$ gives remainder 6.
311 excess 5; $3 + 1 + 1 = 5$ and $5 \div 9$ gives remainder 5.
 $2 + 9 = 11$ and $11 - 6 = 5$. What is your conclusion?

EXERCISES

Make the following subtractions and check:

- | | | | | |
|------------|-------------|-------------|-------------|-------------|
| 1. 564 | 3. 7284 | 5. 7283 | 7. 4312 | 9. 2913 |
| <u>385</u> | <u>3657</u> | <u>6594</u> | <u>2738</u> | <u>1536</u> |
| 2. 783 | 4. 2345 | 6. 3245 | 8. 4945 | 10. 3124 |
| <u>475</u> | <u>1652</u> | <u>1876</u> | <u>3586</u> | <u>1568</u> |

11. Henry has read 32 pages in his book. How many more must he read before he has read 67 pages? 75 pages? 86 pages?

12. Hold a number contest in addition and subtraction.

14. Solution of Problems.—Every problem consists of two parts: the numbers that are given and the number that is to be found. In solving problems use the following directions:

1. *Read the problem carefully.*
2. *Decide what is asked for.*
3. *See how this number is connected with the numbers given.*
4. *Carry out the necessary operations with the given numbers to get the number asked for.*

Apply these directions to the following problems:

1. John earns 50 cents per day and Harry 75 cents. How much do they earn together in 4 days?

2. A train starts from a station going south at the rate of 30 mi. per hour at the same time that another train starts north at the rate of 45 mi. per hour. How far apart will the trains be at the end of 3 hours?

3. In a junior high school one fall, football suits were bought for \$ 8.75 each and head protectors for \$ 2.85 each. What was the total cost to provide this equipment for 13 players?

Sometimes problems seem dissimilar which are in fact alike, because the number asked for is found in just the same way in each problem. Show that the above problems are alike because they are solved by the same process.

EXERCISES

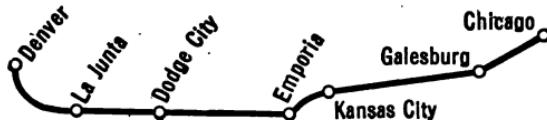
1. A merchant's daily sales for a week were \$ 215, \$ 367, \$ 198, \$ 207, \$ 385, \$ 427. What were his sales for the week?
2. It is 169 mi. from A to B; 207 mi. from B to C; 318 mi. from C to D. How far is it from A to C? How far is it from B to D? How far is it from A to D?

3. What number added to 465 will give 938?

The sum of the number represented by ??? and 465 is 938. How can you **undo this addition** so as to find the number represented by ???? Often we cannot see what principle to use in solving a problem, but we do see the connection between the known number and the one to be found. Use this plan whenever you can. Solve Exs. 4, 5, and 6 in this way.

$$\begin{array}{r} 465 \\ - ??? \\ \hline 938 \end{array}$$

4. The difference between 58 and a smaller number is 29. What is the smaller number?
5. The difference between two numbers is 37. If one number is 19 what is the other number?
6. The difference between two numbers is 56. If one number is 76 what **may** the other number be?
7. If you know one of two numbers and their sum, how can you find the other number? Show that this is the opposite of addition.



8. Find the distance from Kansas City to Denver; from Emporia to La Junta; from Galesburg to Denver.

From Chicago to	
Galesburg.....	177 mi.
Kansas City....	451 mi.
Emporia.....	578 mi.
Dodge City.....	798 mi.
La Junta.....	991 mi.
Denver	1,174 mi.

9. How far will you travel in going from Chicago to Denver and back to Dodge City? From Chicago to Denver and back to Emporia?

10. Is it farther from Galesburg to Dodge City or from Emporia to Denver and how much?

11. How can you find the distance between any two of the cities mentioned above?

15. Multiplication.—Accurate rapid work in multiplication requires that you memorize the thirty-six multiplication combinations thoroughly. Each combination must be known so thoroughly that 6×7 is 42 as quickly as d-o-g is dog.

Give the products in place of the sums on page 6.

16. Short Cuts.—Which can you find the more easily of the products $5 \times 9 \times 6$, $6 \times 5 \times 9$, $9 \times 6 \times 5$, $6 \times 9 \times 5$, $5 \times 6 \times 9$, $9 \times 5 \times 6$? If you have three or more numbers to multiply together, be always on the lookout to find the simplest order to carry out the work.

EXERCISES

Perform the following multiplications without using pencil and paper:

- | | | |
|--------------------------|----------------------------|-----------------------------|
| 1. $2 \times 8 \times 5$ | 5. $9 \times 4 \times 5$ | 9. $50 \times 19 \times 2$ |
| 2. $3 \times 4 \times 5$ | 6. $15 \times 2 \times 7$ | 10. $45 \times 3 \times 2$ |
| 3. $6 \times 7 \times 5$ | 7. $50 \times 17 \times 2$ | 11. $25 \times 37 \times 4$ |
| 4. $4 \times 7 \times 3$ | 8. $35 \times 9 \times 2$ | 12. $4 \times 9 \times 3$ |

13. What will John earn working 6 da. per week for 7 wk. at 50¢ per day?

14. The fare from one city to another is 86¢. What will be the cost of 5 return trip tickets? of 50?

15. Harry delivers papers to 60 customers. If he earns 5¢ per week from each customer, what will he earn in 2 wk.? in 4 wk.? in 8 wk.?

16. A dealer makes 4¢ on every note-book he sells. How much does he make in selling 5 doz.? 3 doz.?

17. In buying canned goods by the dozen from a grocer, 2¢ is saved on each can. How much is saved in buying 6 doz.? 10 doz.? 5 doz.?

17. Long Multiplication.—First, multiply 600, 30, and 7 each by 5 giving 3,000, 150, and 35 which added give 3,185. Second, multiply the same numbers by 20 which added give 12,740. In practice the 0 is usually omitted. Third, multiply the same numbers by 400 which added give 254,800, the last two 0's again being omitted in practice.

Note that we multiply by 5, 20, and 400; not by 5, 2, and 4. Finally, adding the numbers found by multiplication we have 270,725. Read these large numbers.

18. Checking Multiplication.—To test your work multiply the excess over 9 of the multiplicand by the excess of the multiplier. Then find the excess of this number. If the multiplication has been carried out correctly, the excess just found and that of the product of the two numbers are equal.

The excess of 637 is 7 and of 425 is 2. $2 \times 7 = 14$, whose excess is 5. The excess of 270,725 is also 5. What is your conclusion?

EXERCISES

Read the following numbers as suggested before, carry out the multiplications as rapidly as possible, and check the results:

1. 548	4. 637	7. 5683	10. 8459	13. 4896
<u>213</u>	<u>583</u>	<u>574</u>	<u>4896</u>	<u>6087</u>
2. 604	5. 567	8. 8579	11. 6807	14. 6805
<u>374</u>	<u>794</u>	<u>709</u>	<u>7468</u>	<u>7306</u>
3. 693	6. 748	9. 6784	12. 7485	15. 7406
<u>486</u>	<u>586</u>	<u>860</u>	<u>5386</u>	<u>6408</u>

16. Play a number game on addition, subtraction, and multiplication examples.

19. Division.—Division is the opposite of multiplication, just as subtraction is the opposite of addition. In $42 \div 6$ we ask, "How many 6's are there in 42"? or "What number multiplied by 6 gives 42"? With $6 \times 7 = 42$ and $7 \times 6 = 42$ must be memorized and connected thoroughly that $42 \div 6 = 7$, $42 \div 7 = 6$, $\frac{1}{6}$ of 42 = 7, and $\frac{1}{7}$ of 42 = 6.

EXERCISES

1. How many are there in 8 piles of apples having 6 in each pile? having 9 in each pile? having 7 in each pile?
2. How many piles of apples with 7 in each can be made from 42 apples? from 35 apples? from 63 apples?
3. Into how many groups of 6 in each can you divide 42? 36? 120? 600? 300? 54? 30? 240?
4. How many are there in each group of 8 groups formed from 48? from 72? from 40? from 160? from 240?
5. By what must you multiply 9 to get 45? 12 to get 60? 10 to get 240? 9 to get 63? 8 to get 72? 21 to get 42?

Supply the missing numbers in the following:

- | | |
|-------------------------|---------------------------------|
| 6. $6 \times ? = 48$ | 16. $? \times 5 = 60$ |
| 7. $5 \times ? = 40$ | 17. $13 \times ? = 39$ |
| 8. $9 \times ? = 72$ | 18. $30 \times ? = 270$ |
| 9. $? \times 4 = 8$ | 19. $150 \div ? = 50$ |
| 10. $7 \times ? = 63$ | 20. $25 \times ? = 75$ |
| 11. $8 \times ? = 56$ | 21. $2 \times 7 \times 5 = ?$ |
| 12. $150 \div ? = 30$ | 22. $2 \times 5 \times ? = 40$ |
| 13. $? \times 10 = 430$ | 23. $5 \times 17 \times 2 = ?$ |
| 14. $17 \times ? = 34$ | 24. $4 \times 91 \times 25 = ?$ |
| 15. $48 \times ? = 144$ | 25. $4 \times 7 \times 25 = ?$ |

20. Short Division.—Use short division whenever the divisor is small.

$$\begin{array}{r} 243 - 1 \\ \hline 4) 973 \end{array}$$
 4 is contained in 9 hundred, 2 hundred times and 1 hundred over. The 1 hundred added to the 7 tens give 17 tens. Why? 4 is contained in 17 tens, 4 tens times and 1 ten over. The 1 ten and the 3 give 13 ones, or units. 4 is contained in the 13 units, 3 units times and 1 over. Each digit of the quotient is written above the corresponding one in the dividend.

21. Long Division.—Long Division is used when the divisor is large.

Quotient	<u>428</u>	Dividend	62 is contained in 265 hundred 4 hundred times, with a remainder of 17 hundred. These 17 hundred with the 8 tens make 178 tens. 62 is contained in 178 tens, 2 tens times, with a remainder of 54 tens. The 54 tens and 3 units make 543 units. 62 is contained in 543 units, 8 units times, with a remainder of 47.
Divisor	<u>62)</u>		
	<u>26583</u>		
	<u>248</u>		
	<u>178</u>		
	<u>124</u>		
	<u>543</u>		
	<u>496</u>		
Remainder	<u>47</u>		

When the second digit of the divisor is small, as in 62 or 716, the first digit can generally be used alone to decide each figure in the quotient. When the second digit is large, as in 68 or 382, the first digit increased by one can be used to decide upon each digit in the quotient.

22. Checking Division.—The quotient times the divisor added to the remainder equals the dividend, if the division is correct. Here is a better plan: Multiply the excess of 9's of the divisor and quotient together. To this add the excess of the remainder. The excess of this last number equals the excess of the dividend if the work is correct.

Excess of 62 is 8 and of 428 is 5. Their product is 40. Excess of 47 is 2 which added to 40 gives 42. The excess of 42 is 6, the same as of 26,583. What is your conclusion?

EXERCISES

Read the following, carry out the divisions as shown, and test your results.

- | | | |
|-----------------|---------------------|-----------------------|
| 1. $674 \div 3$ | 7. $5,408 \div 39$ | 13. $10,306 \div 406$ |
| 2. $534 \div 8$ | 8. $6,057 \div 83$ | 14. $34,027 \div 256$ |
| 3. $706 \div 7$ | 9. $2,056 \div 35$ | 15. $42,071 \div 405$ |
| 4. $965 \div 5$ | 10. $6,304 \div 54$ | 16. $63,405 \div 354$ |
| 5. $842 \div 9$ | 11. $3,225 \div 67$ | 17. $12,064 \div 519$ |
| 6. $308 \div 6$ | 12. $2,709 \div 48$ | 18. $20,301 \div 706$ |

19. Divide four thousand, three hundred sixty-nine by three hundred ninety-four.

20. Divide eleven thousand, fifty-four by six hundred nineteen.

21. Divide twenty thousand, one hundred five by one hundred seventy-two.

22. Copy and add each column on page 5.

Add, subtract, and multiply each of the following sets of numbers. Use pencil and paper only when necessary.

- | | | | | | | | | | |
|-----|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 23. | <u>57</u> | <u>64</u> | <u>73</u> | <u>85</u> | <u>93</u> | <u>65</u> | <u>35</u> | <u>47</u> | <u>84</u> |
| | <u> 9</u> | <u> 8</u> | <u> 9</u> | <u> 8</u> | <u> 9</u> | <u> 7</u> | <u> 8</u> | <u> 9</u> | <u> 8</u> |
| 24. | <u>34</u> | <u>27</u> | <u>43</u> | <u>62</u> | <u>35</u> | <u>23</u> | <u>48</u> | <u>63</u> | |
| | <u> 8</u> | <u> 9</u> | <u> 9</u> | <u> 7</u> | <u> 8</u> | <u> 9</u> | <u> 8</u> | <u> 9</u> | |
| 25. | <u>15</u> | <u>32</u> | <u>53</u> | <u>62</u> | <u>74</u> | <u>68</u> | <u>79</u> | <u>28</u> | |
| | <u> 8</u> | <u> 9</u> | <u> 7</u> | <u> 9</u> | <u> 5</u> | <u> 4</u> | <u> 3</u> | <u> 5</u> | |
| 26. | <u>19</u> | <u>28</u> | <u>37</u> | <u>29</u> | <u>38</u> | <u>17</u> | <u>29</u> | <u>18</u> | |
| | <u> 5</u> | <u> 6</u> | <u> 5</u> | <u> 4</u> | <u> 5</u> | <u> 8</u> | <u> 6</u> | <u> 3</u> | |

23. Divisors: Composite Numbers.—Numbers which are made up from two or more numbers multiplied together, as 15 or 30, are called **composite numbers**. The numbers which are multiplied together are called **divisors** or **factors** of the larger number. The divisors of 15 are 3 and 5. There are four sets of divisors in 30: 2 and 15, 3 and 10, 5 and 6, or 2, 3, and 5.

The following rules will be helpful in finding divisors of numbers:

1. *All even numbers are divisible by 2.*
2. *If that part of the number formed by the last two digits is divisible by 4, the entire number is divisible by 4. Since 28 is divisible by 4 then 32,728 is divisible by 4. Can you divide the following by 4: 3,248 ? 745 ? 1,724 ?*
3. *If that part of the number formed by the last three digits is divisible by 8, the entire number is divisible by 8. Thus, 753,296 is divisible by 8 because 296 is divisible by 8. Test the following for divisibility by 8: 24,168, 7,456, 90,816, 35,479.*
4. *All numbers ending in 5 or 0 are divisible by 5.*
5. *All numbers ending in 0 are divisible by 10.*
6. *If the sum of the digits of a number is divisible by 9, the entire number is divisible by 9. The sum of the digits of 34,956 is 27, hence divisible by 9, therefore 34,956 is divisible by 9. Which of the following are divisible by 9: 24,795 ? 681,453 ? 745,613 ? 24,738 ?*
7. *If the sum of the digits of a number is divisible by 3, the number is divisible by 3. Apply this to 246, 753, 842, 8,574, 3,847, 5,247.*
8. *Many useful combinations arise from the rules just stated. A number ending in 5 or 0 whose digits add up to a sum divisible by 3 is divisible by both 5 and 3. Why? Hence it is divisible by 15. Why?*

24. Prime Numbers.—A number which has no factor but itself and one is called a **prime number**. Such numbers are 7, 19, 347, etc.

EXERCISES

1. Which of the following numbers are prime and which are composite: 7, 6, 15, 34, 29, 245, 111, 214, 762, 648, 830, 948, 19, 459, 724, 245, 215, 624, 495, 34,832?
2. Which of the numbers in Ex. 1 are divisible by 2? by 3? by 4? by 5? by 6? by 8? by 9?
3. By what prime numbers must a number be divisible if it is divisible by 6? by 4? by 10? by 15? by 12?
4. How can you tell if a number is divisible by 6? by 12? by 15? by 18? by 45?
5. Apply the tests worked out in Ex. 4 to Ex. 1 and also to the following numbers: 942, 924, 435, 4,356, 3,465, 5,364, 6,129, 6,912, 528, 582.
6. Find the prime divisors of the first six numbers in Ex. 5.

25. Short Cuts.—Divisors can often be used to simplify multiplication. The price of 160 acres of land at \$45 per acre will be 160×45 , or $80 \times 2 \times 45$. By carrying out the last multiplication first it becomes 80×90 , which is mentally found to be \$7,200. We are here using the principle learned on page 15 about using the simplest order of multiplication.

EXERCISES

Do not use pencil and paper in the following:

1. Find the price of 80 A. of land at \$45 per acre; of 40 A. at \$35 per acre; of 160 A. at \$55 per acre.
2. Find the price of 8 doz. oranges at 45¢ per dozen; of 12 doz. lemons at 35¢ per dozen.
3. Henry drives his automobile at the rate of 35 mi. per hour. How far does he go in 8 hr.? in 12 hr.? in 6 hr.?

4. Richard raised Belgian hares, which he sells at 35¢ each. How much does he receive for 4? for 6? for 8? for 12?

5. Glen has raised lima beans, which he sells at 6¢ per quart. What will he receive for 15 qt.? for 25 qt.? for 35 qt.?

6. At 28¢ per dozen find the price of 5 doz. eggs; of 15 doz.; of 6 doz.; of 25 doz.

7. Play a number game, using multiplication and division examples.

26. Multiplication by 10, 100, etc.—To multiply any whole number by 10, 100, etc., we need only annex as many zeros as there are zeros in the multiplier. Thus, $567 \times 100 = 56,700$. Multiplication by 20, 30, etc., can also be simplified, thus, $317 \times 20 = 317 \times 2 \times 10 = 634 \times 10 = 6,340$.

EXERCISES

Find the following products without the use of pencil and paper:

1. 546×100	4. 625×30	7. 532×60
2. 125×20	5. 315×300	8. 413×50
3. 413×40	6. 256×200	9. 752×40

27. Aliquot Parts.—Divisors of 10 are 2 and 5. Divisors of 100 are 2, 5, 10, 20, 25, 50. These divisors are generally referred to as **aliquot parts** of 10 and 100. Multiplication by 10 and division by 2 will produce the same result as multiplication by 5. Try it with 456, 324, and 8,462. To multiply by 10 a zero is merely annexed, while division by 2 is simpler than multiplication by 5. To multiply by 25 annex two zeros and divide by 4. Why? To multiply by 50 annex two zeros and divide by 2. Why? Learn to multiply in this manner by 5, 25, and 50 without the use of pencil and paper.

EXERCISES

1. Make the following multiplications in the usual way, and as on page 22—see how long each takes: 546×5 , 326×5 , $4,236 \times 5$, $1,482 \times 5$, $3,046 \times 5$.
2. Perform the following multiplications as for Ex. 1: 732×25 , 426×5 , $4,213 \times 50$, $4,026 \times 25$, $3,045 \times 50$.
3. At 25¢ per yard find the cost of 12 yd. of cloth; of 17 yd.; of 46 yd.; of 119 yd.; of 29 yd.
4. At 32¢ per yard find the cost of 5 yd. of cloth; of 10 yd.; of 25 yd.; of 50 yd.
5. Find the cost at 25¢ per dozen of 12 doz. eggs; of 16 doz.; of 32 doz.
6. Find the cost at \$5 per ton of 18 T. coal; of 36 T.; of 56 T.; of 42 T.
7. How far will an automobile go at the rate of 25 mi. per hour in 12 hr. ? in 16 hr. ? in 56 hr. ?
8. Where, out of school, have you noticed multiplication that could be simplified as in Arts. 26 and 27. Carry out some of them.
28. **Powers.**—In the place of 7×7 , is written 7^2 ; in the place of $5 \times 5 \times 5$ is written 5^3 , etc. We say that 7 has been raised to the **2nd power**; that 5 has been raised to the **3rd power**, and so on. Roots will be studied later.

EXERCISES

Give the values of the following:

- | | | | | |
|----------|----------|-----------|------------|------------|
| 1. 4^2 | 5. 4^4 | 9. 5^4 | 13. 25^2 | 17. 12^3 |
| 2. 5^3 | 6. 2^4 | 10. 6^3 | 14. 14^2 | 18. 4^3 |
| 3. 3^4 | 7. 4^2 | 11. 9^2 | 15. 23^2 | 19. 2^6 |
| 4. 8^2 | 8. 2^3 | 12. 3^4 | 16. 32^2 | 20. 15^2 |

29. Signs of Operation.—If several signs of operation occur in the same expression, as in

$$12 + 27 \div 3^2 \times 4 + 36 \times 5 \div \frac{3}{4} \text{ of } 2^3 - 8 \times 3^2 \div \frac{3}{5} \text{ of } 20,$$

they must be carried out in the following order:

1. *Raise to powers and extract roots.*
2. *Multiply as indicated by "of."*
3. *Multiply and divide in the order in which they arise.*
4. *Add and subtract in any order.*

The above expression would hence be:

$$\begin{aligned} 12 + 27 \div 9 \times 4 + 36 \times 5 \div \frac{3}{4} \text{ of } 8 - 8 \times 9 \div \frac{3}{5} \text{ of } 20 &= \\ 12 + 27 \div 9 \times 4 + 36 \times 5 \div 6 - 8 \times 9 \div 12 &= \\ 12 + 3 \times 4 + 180 \div 6 - 72 \div 12 &= \\ 12 + 12 + 30 - 6 &= 48 \end{aligned}$$

30. Parentheses.—Operations placed inside of parentheses, (), must be carried out before any other operations. Thus, $5 \times (3 + 7 - 4) = 5 \times 6 = 30$. How?

EXERCISES

Carry out the following operations:

- | | |
|--|--|
| 1. $7 + 6 \times 2 \div 4$ | 9. $24 \div \frac{2}{3} \text{ of } 6 - 5 + 4^2$ |
| 2. $9 + 12 \div 4 \times 3$ | 10. $5 + 16 \div \frac{2}{5} \text{ of } 10$ |
| 3. $5 + 3 \times (5 - 2)$ | 11. $56 \div (2 \times 7) + 3 \times (5 + 7)$ |
| 4. $8 - 12 \div \frac{2}{3} \text{ of } 6$ | 12. $6^2 \div 4 - 15 \times 6 \div 45$ |
| 5. $3^3 \div 9 + 17$ | 13. $10^2 \div \frac{2}{3} \text{ of } 15 - (12 - 8)$ |
| 6. $45 \div 3^2 - 5$ | 14. $7 \times 2^2 \div 14 + (5 + 7) \div 6$ |
| 7. $36 \div (4 + 5)$ | 15. $18 \div 3^2 + \frac{2}{3} \text{ of } 30 \div 20$ |
| 8. $(29 + 16) \div 3^2$ | 16. $\frac{2}{3} \text{ of } 6^2 \div 4 + 15^2 \div 5^2$ |

17. Play a number game, using multiplication and division.

31. Solution of Problems.—Division is the opposite of multiplication, just as subtraction is the opposite of addition. This fact can often be used in the solution of problems.

1. How many hours will it take a train to go 448 mi. at the rate of 32 mi. per hour?

How can you find how far a train goes in any number of hours at 32 mi. per hour? In the multiplication ?? stands for the number of hours that it takes the train to go 448 mi. How can you undo the multiplication so as to find ??, the number of hours ??
How many hours is this? Sometimes we could solve a problem
if one of the known numbers was asked for. Then show by the
use of question-marks how that known number is found. Next find
the number for which the question-marks stand.

EXERCISES

Solve as many of the following problems without the use of pencil and paper as possible:

1. How far will an automobile go in 12 hr. if it goes 10 mi. per hour? 15 mi. per hour? 30 mi. per hour?
2. How long will it take an automobile to go 360 mi. at the rate of 10 mi. per hour? 25 mi. per hour? 12 mi. per hour?
3. Find the rate of an automobile which goes 240 mi. in 10 hr.; in 8 hr.; in 12 hr.; in 15 hr.; in 16 hr.
4. Write out each statement in problems 2 and 3 as a multiplication, placing a question-mark for the unknown number. Then show how to find the number which put in the place of the question-mark gives the correct multiplication.
5. What is the connection between the rate of motion, distance travelled, and the time of going?
6. A certain number multiplied by 315 equals 4,410. What is the number?

7. Supply numbers for the question-marks so as to make true multiplications in the following:

215	215	???	382	1932
25	?	12	?	225
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
???	1505	708	5730	??????

8. What number multiplied by 235 equals 6,110 ?
9. At 5¢ apiece, what does John receive for 14 Saturday Evening Posts ? for 18 ?
10. At \$15 per ton, what is the cost of 14 T. of hay ? of 18 T. ? of 24 T. ? of 25 T. ?
11. How many miles per hour does an automobile travel if it goes 64 mi. in 4 hr. ? 120 mi. in 6 hr. ? 115 mi. in 5 hr. ?
12. How many nuts should each of 3 boys get if they gather 12 qt. ? of 5 boys if they gather 40 qt. ? of 7 boys if they gather 98 qt. ?
13. Show that problems 9 and 10 are just the reverse of problems 11 and 12.
14. What number multiplied by 7 will give 245 ?
15. The product of 15, 18, and a third number is 3,780. Find the third number.
16. What does a man receive per hour who is paid \$19.20 per week for working 8 hr. per day ?
17. The sum of 857, 694, and a third number is 3,326. Find the third number.
18. George Washington died in 1799 at the age of 67 yr. In what year was he born ?
19. The battle of the Monitor and the Merrimac occurred in 1862 when John Ericsson, the inventor of the Monitor, was 59 yr. old. In what year was he born ?

20. Elias Howe invented the sewing-machine in 1845. How long ago was that? He died 22 yr. later. How long ago was that?

21. A merchant's daily sales for a week were \$ 135.78, \$ 254.15, \$ 143.35, \$ 815.34, \$ 147.85, and \$ 298.76. His expenses were \$ 208.67, \$ 68.47, \$ 59.65, \$ 74.93, \$ 178.45, and \$ 57.63. Find the "net sales," that is, the amount of sales over expenditures.

22. The cashier of a store begins the day's business with \$ 85.76; sales amount to \$ 139.48; expenditures are \$ 79.64. What cash should there be on hand?



23. A cashier began the day with \$ 94.68; sales were \$ 176.85; expenditures were \$ 86.74. Had change been made correctly if the cash on hand at the end of the day was \$ 184.68?

24. A cashier began the day with \$ 139.05; \$ 48.67 was paid out during the day and the cash on hand at the end of business was \$ 247.39. What amount of sales should the cash register show?

25. A store began the day's business with \$ 248.57; sales were \$ 176.35; expenditures were \$ 97.86; \$ 250 was deposited in the bank. What should be the cash on hand?

26. Explain fully the connection between cash on hand at the beginning of a day's business, sales, money paid out, and cash on hand at the end of the day.

27. Play a number game, using simple problems in whole numbers.

II

COMMON FRACTIONS

32. Meaning of Fractions.—Any part of an object or of a group of objects is called a **fraction**. One of the two equal parts into which an apple is divided is one-half of an apple, while six apples are one-half of a dozen apples.



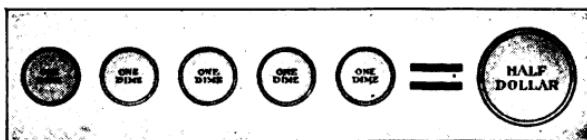
Common fractions are written thus, $\frac{1}{2}$. The number below the line is called **denominator** and tells into how many equal parts the object or group of objects is divided; it names the fraction with which we are dealing. The number above the line is called **numerator** and tells how many of the equal parts of an object or group of objects the fraction represents; it is the numberer of the fraction. The numerator and denominator are called the **terms**.

33. Integers.—Whole numbers are also called **integers**.

EXERCISES

1. What part of a foot is 3 in.? 6 in.? 9 in.?
2. In a foot there are how many $\frac{1}{2}$ in.? $\frac{1}{4}$ in.? $\frac{1}{5}$ in.?
3. A quarter is what part of \$2? of \$5? of \$1.25?
4. A dime is what part of \$1? of \$3? of \$1.60?
5. What part of a dozen are 6 objects? 8 objects?

34. Equivalent Fractions.—Fractions of the same value may have different numerators and different denominators. Such fractions are called **equivalent fractions**. One of the 2 equal parts of an object and 5 of 10 equal parts of the same object represent the same amount. Hence, $\frac{1}{2} = \frac{5}{10}$.



Similarly, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$, etc. How?

35. Reduction of Fractions.—Fractions may be reduced to equivalent fractions by multiplying or dividing both terms by the same number. Why?

Thus, $\frac{6}{8} = \frac{3}{4}$; $\frac{6}{18} = \frac{1}{3}$; $\frac{9}{15} = \frac{3}{5}$; $\frac{9}{15} = \frac{3}{5}$.

36. Lowest Terms.—Fractions whose terms have no common divisor are said to be in their **lowest terms**. Reduce all final fractions to their lowest terms.

EXERCISES

Use pencil and paper only when necessary.

1. Show by objects that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$; $\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$; $\frac{1}{5} = \frac{2}{10} = \frac{3}{15}$.
2. Give three fractions equivalent to each of the following: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$.
3. Reduce to their lowest terms: $\frac{4}{6}$, $\frac{5}{35}$, $\frac{6}{18}$, $\frac{7}{31}$, $\frac{9}{15}$, $\frac{8}{20}$, $\frac{10}{15}$, $\frac{14}{21}$, $\frac{22}{33}$, $\frac{17}{27}$.
4. Reduce the results found in exercises on page 28 to their lowest terms.
5. Which is the larger, $\frac{1}{3}$ or $\frac{2}{5}$? $\frac{4}{5}$ or $\frac{3}{5}$?
6. Change $\frac{4}{5}$ and $\frac{9}{10}$ so that you can see which is the larger. Also compare $\frac{2}{3}$ and $\frac{3}{5}$; $\frac{2}{3}$ and $\frac{4}{5}$; $\frac{3}{4}$ and $\frac{5}{6}$; $\frac{1}{2}$ and $\frac{1}{3}$.

37. Addition and Subtraction of Fractions.—Fractions having the same denominator may be added or subtracted at once. To add $\frac{3}{8}$ and $\frac{1}{8}$ is the same as adding 3 apples and 1 apple. In the first case we get $\frac{4}{8}$ and in the second we get 4 apples. Fractions having different denominators must be changed to equivalent ones so that all will have the same denominator.

$$\text{Thus, } \frac{3}{4} + \frac{1}{8} = \frac{6}{8} + \frac{1}{8} = \frac{7}{8}.$$

$$\text{Similarly, } \frac{1}{4} - \frac{1}{16} = \frac{4}{16} - \frac{1}{16} = \frac{3}{16}. \text{ How?}$$

EXERCISES

Carry out the first 16 exercises without the use of pencil and paper.

1. $\frac{3}{5} + \frac{1}{5}$

9. $\frac{1}{2} + \frac{3}{4}$

17. $\frac{3}{4} + \frac{2}{3}$

2. $\frac{1}{7} + \frac{5}{7}$

10. $\frac{1}{3} - \frac{1}{6}$

18. $\frac{2}{15} + \frac{3}{10}$

3. $\frac{4}{5} - \frac{3}{5}$

11. $\frac{3}{4} + \frac{1}{8}$

19. $\frac{1}{5} + \frac{1}{6}$

4. $\frac{7}{8} - \frac{1}{2}$

12. $\frac{5}{6} - \frac{2}{3}$

20. $\frac{7}{8} - \frac{2}{3}$

5. $\frac{2}{3} - \frac{1}{6}$

13. $\frac{4}{5} - \frac{3}{10}$

21. $\frac{5}{6} - \frac{3}{8}$

6. $\frac{3}{4} + \frac{1}{2}$

14. $\frac{4}{5} - \frac{1}{5}$

22. $\frac{2}{3} + \frac{5}{12}$

7. $\frac{1}{3} + \frac{5}{6}$

15. $\frac{1}{8} + \frac{1}{16}$

23. $\frac{3}{7} + \frac{3}{4}$

8. $\frac{7}{8} - \frac{5}{16}$

16. $\frac{2}{3} + \frac{1}{6}$

24. $\frac{2}{3} + \frac{3}{8}$

25. $\frac{1}{2} + \frac{3}{4} - \frac{5}{8}$

28. $\frac{3}{8} + \frac{1}{5} - \frac{3}{20}$

31. $\frac{1}{5} + \frac{2}{3} - \frac{7}{20}$

26. $\frac{3}{8} + \frac{5}{12} - \frac{7}{10}$

29. $\frac{5}{6} - \frac{2}{3} + \frac{3}{4}$

32. $\frac{3}{8} - \frac{7}{20} + \frac{8}{45}$

27. $\frac{7}{15} + \frac{3}{4} + \frac{9}{10}$

30. $\frac{7}{12} - \frac{1}{3} + \frac{3}{8}$

33. $\frac{3}{8} - \frac{1}{10} + \frac{7}{15}$

34. If the sum of two fractions is $\frac{11}{12}$ and one fraction is $\frac{3}{8}$, what is the other fraction?

35. What fraction added to $\frac{3}{8}$ gives $\frac{11}{12}$?

36. What fraction subtracted from $\frac{3}{4}$ gives $\frac{1}{10}$?

37. In smoothing off a brass cylinder for a machine it was reduced from a diameter of $\frac{3}{4}$ in. to $\frac{11}{12}$ in. How much smaller is the second diameter?

38. Mixed Numbers.—A whole number and a fraction taken together are called a **mixed number**. Mixed numbers are written with the fraction following right after the whole number; $6\frac{1}{4}$, $5\frac{3}{4}$, etc.

39. Proper and Improper Fractions.—In **proper fractions** the numerator is less than the denominator; $\frac{3}{5}$. In **improper fractions** the numerator is not less than the denominator; $\frac{8}{3}$ or $\frac{5}{2}$.

40. Reducing Mixed Numbers to Improper Fractions.—The whole number in a mixed number can be reduced to a fraction and added to the fraction of the mixed number.

$$\text{Thus, } 2\frac{1}{4} = \frac{1}{4} + \frac{8}{4} = \frac{18}{4}.$$

41. Reducing Improper Fractions to Mixed Numbers.—An improper fraction can be reduced to a mixed number.

$$\text{Thus, } \frac{8}{3} = \frac{6}{3} + \frac{2}{3} = 1\frac{2}{3}.$$

Reduce all final results to those in which the fractions are proper fractions in their lowest terms.

EXERCISES

Use pencil and paper only when absolutely necessary.

1. Simplify: $\frac{12}{5}$, $\frac{12}{8}$, $\frac{18}{6}$, $\frac{30}{8}$, $\frac{32}{4}$, $\frac{15}{5}$, $\frac{14}{4}$.
2. Change to improper fractions: $2\frac{1}{3}$, $3\frac{1}{5}$, $6\frac{2}{3}$, $7\frac{1}{3}$, $6\frac{3}{4}$, $8\frac{1}{7}$, $7\frac{2}{5}$, $9\frac{2}{3}$, $6\frac{1}{8}$, $4\frac{1}{4}$.
3. How many dollars and quarters are there in 8 quarters? in 10 quarters? in 6 quarters? in 15 quarters?
4. Express the above in dollars and fractions of a dollar, as 7 quarters equal $1\frac{3}{4}$ dollars.
5. Express as feet and fractions of a foot: 15 in.; 18 in.; 24 in.; 32 in.; 42 in.; 48 in.; 52 in.
6. Into how many groups and fractions of a group of 6 in each, can you divide 12? 15? 18? 26? 33? 27? 38?

42. Addition of Mixed Numbers.

$14\frac{3}{4}$	$\frac{1}{2}\frac{5}{6}$	Change fractions to equivalent fractions with the same denominator and place them to the right.
$8\frac{2}{5}$	$\frac{8}{20}$	Add fractions. Reduce to a mixed number if the sum is an improper fraction. Add the whole numbers and any ones found in the sum of the fractions.
23		
$26\frac{1}{2}$	$\frac{1}{2}\frac{6}{6}$	
$72\frac{1}{2}\frac{3}{6} = 1\frac{1}{2}\frac{3}{6}$	$\frac{3}{6}$	Annex the remaining fraction to this sum.

EXERCISES

Read and add the following:

1. $457392\frac{1}{2}$	3. $678409\frac{2}{3}$	5. $845673\frac{5}{6}$	7. $456793\frac{7}{8}$
$876908\frac{1}{3}$	$846936\frac{3}{5}$	$487563\frac{3}{4}$	$756828\frac{3}{4}$
$376593\frac{5}{6}$	$293847\frac{8}{10}$	$835674\frac{1}{8}$	$546792\frac{5}{6}$
$750869\frac{7}{4}$	736895	$645897\frac{1}{3}$	$435683\frac{7}{12}$
2. $647083\frac{4}{5}$	4. $467538\frac{2}{5}$	6. $457308\frac{3}{5}$	8. $560789\frac{3}{8}$
$846795\frac{3}{5}$	$39567\frac{3}{4}$	98476	$3567084\frac{2}{5}$
$670459\frac{1}{2}$	$106705\frac{2}{3}$	$48675\frac{1}{6}$	$4706893\frac{7}{10}$
645094	$467538\frac{5}{6}$	$754698\frac{8}{10}$	45679
$378456\frac{5}{6}$	$45739\frac{8}{10}$	$687459\frac{1}{5}$	$5674985\frac{1}{4}$

43. Subtraction of Mixed Numbers.—In the subtraction of mixed numbers, consider the fractions first just as in addition.

$35\frac{2}{3}$	$\frac{1}{2}\frac{1}{2}$	Change $\frac{2}{3}$ and $\frac{1}{4}$ to $\frac{8}{6}$ and $\frac{3}{6}$. $\frac{8}{6}$ added to $\frac{5}{6}$
$12\frac{1}{4}$	$\frac{1}{2}\frac{1}{2}$	equals $\frac{8}{6}$; write $\frac{5}{6}$. 12 added to 23 equals 35. Write
$23\frac{5}{6}$	$\frac{5}{6}$	23 and annex $\frac{5}{6}$.

The fraction in the subtrahend may be larger than that in the minuend.

$64\frac{1}{5}$	$\frac{4}{6}$	Change $\frac{1}{5}$ and $\frac{4}{6}$ to $\frac{4}{20}$ and $\frac{16}{20}$. $\frac{16}{20}$ added to $\frac{9}{20}$
$21\frac{3}{4}$	$\frac{15}{20}$	equals $\frac{24}{20}$, or $1\frac{4}{20}$; write $\frac{9}{20}$. 1 added to 21 and 42
$42\frac{9}{20}$	$\frac{9}{20}$	make 64. Write 42 and annex $\frac{9}{20}$. Hence, $21\frac{3}{4}$ added to $42\frac{9}{20}$ make $64\frac{1}{2}$.

EXERCISES

Find:

1. $\frac{2}{3} + \frac{5}{6} - \frac{1}{12}$	4. $\frac{3}{4} + \frac{2}{3} - \frac{5}{6}$	7. $\frac{7}{8} + \frac{5}{6} + \frac{7}{12}$
2. $\frac{1}{5} + \frac{3}{4} + \frac{7}{10}$	5. $\frac{1}{6} + \frac{3}{4} - \frac{11}{12}$	8. $\frac{3}{8} - \frac{1}{4} + \frac{11}{12}$
3. $\frac{2}{5} + \frac{1}{3} + \frac{7}{15}$	6. $\frac{5}{6} + \frac{3}{4} + \frac{2}{21}$	9. $\frac{2}{15} + \frac{5}{12} + \frac{6}{20}$

Find the following differences:

10. $678\frac{1}{2}$ <u>$435\frac{1}{4}$</u>	13. $6352\frac{5}{8}$ <u>$2584\frac{1}{2}$</u>	16. $468396\frac{5}{8}$ <u>$86382\frac{7}{5}$</u>
11. $57\frac{3}{5}$ <u>$29\frac{1}{4}$</u>	14. $7428\frac{3}{4}$ <u>$5516\frac{7}{8}$</u>	17. $467327\frac{7}{15}$ <u>$240569\frac{3}{20}$</u>
12. $376\frac{5}{8}$ <u>$197\frac{1}{3}$</u>	15. $4623\frac{1}{4}$ <u>$2852\frac{2}{3}$</u>	18. $348239\frac{2}{5}$ <u>$75619\frac{3}{4}$</u>

Find the following sums:

19. $546739\frac{1}{8}$ 35578 $\frac{3}{4}$ 1635827 $\frac{2}{5}$ 45735 $\frac{7}{10}$ <u>735698</u>	20. $568439\frac{4}{7}$ 367285 $\frac{5}{14}$ 46726 526783 $\frac{1}{11}$ <u>45738</u>	21. $734892\frac{5}{8}$ 537815 $\frac{3}{8}$ 248349 $\frac{7}{12}$ 137973 $\frac{9}{8}$ <u>245684$\frac{2}{3}$</u>
--	--	---

22. A man spends $\frac{1}{6}$ of his yearly income for rent, $\frac{1}{4}$ for food, $\frac{1}{2}$ for clothing, and $\frac{1}{8}$ for sundries. What part of his income does he save?

23. John sells 24 Saturday Evening Posts every week. If he sells $\frac{1}{2}$ the first afternoon and $\frac{3}{8}$ the next morning, what part of the total number has he then sold? What part of the total does he yet have to sell? How many papers has John yet to sell?

24. A baseball team won $\frac{3}{5}$ of its games and tied $\frac{1}{10}$ of its games. What part of the games played were lost?

25. Play a number game, using addition and subtraction examples of mixed numbers.

44. Multiplication of Fractions by Whole Numbers.—Five times $\frac{3}{8}$ equals $\frac{15}{8}$, just as 5 times 3 apples equals 15 apples. As final results must all be reduced to their lowest terms, the numerator is multiplied by the whole number and placed over the denominator, **only** when the whole number and denominator have no common factor. If the whole number and the denominator have a common divisor each is divided by this first. Thus:

$$4 \times \frac{3}{16} = \frac{3}{4}; \quad 6 \times \frac{5}{9} = \frac{10}{3} = 3\frac{1}{3}.$$

This process is called **cancellation**.

EXERCISES

Use pencil and paper only when absolutely necessary. Look for divisors and for the simplest order of carrying out the required operations.

- | | | |
|----------------------------|---------------------------------------|---------------------------------------|
| 1. $4 \times \frac{5}{12}$ | 8. $8 \times \frac{5}{12}$ | 15. $4 \times \frac{5}{32} \times 20$ |
| 2. $3 \times \frac{1}{6}$ | 9. $5 \times \frac{4}{15}$ | 16. $\frac{2}{3} \times 15 \times 6$ |
| 3. $5 \times \frac{7}{10}$ | 10. $7 \times \frac{4}{8} \times 6$ | 17. $18 \times \frac{5}{12} \times 2$ |
| 4. $7 \times \frac{8}{14}$ | 11. $\frac{3}{5} \times 7 \times 10$ | 18. $10 \times \frac{7}{15} \times 3$ |
| 5. $8 \times \frac{5}{16}$ | 12. $\frac{3}{10} \times 15 \times 4$ | 19. $6 \times \frac{7}{12} \times 4$ |
| 6. $9 \times \frac{2}{3}$ | 13. $\frac{7}{2} \times 6 \times 10$ | 20. $5 \times \frac{3}{4} \times 12$ |
| 7. $.6 \times \frac{3}{4}$ | 14. $6 \times \frac{3}{15} \times 4$ | 21. $8 \times 5 \times \frac{7}{16}$ |

22. If it takes $\frac{2}{3}$ yd. of cloth to make a hat, how much will be necessary to make 12 hats? 16 hats? 14 hats?

23. Henry can ride $\frac{1}{5}$ mi. a minute on his bicycle. How far can he ride in 40 min.? in 45 min.? in 90 min.?

24. A brick is $\frac{2}{3}$ ft. long. What will be the length in feet of 6 bricks? of 8 bricks? of 15 bricks? of 25 bricks?

25. John earns $\frac{3}{4}$ of a dollar a day. How much will he earn in 12 da.? in 15 da.? in a week? in a month?

45. Multiplication of Fractions by Fractions.—Just as $\frac{1}{2}$ of 6 apples equals 3 apples so $\frac{1}{2}$ of $\frac{6}{5}$ equals $\frac{3}{5}$. If there are 4 apples in one group, 3 of these groups contain $3 \times 4 = 12$ apples. Of used in this manner indicates multiplication. Hence, $\frac{1}{2}$ of 6 and $\frac{1}{2} \times 6$ mean the same and each gives 3. In the same way $\frac{1}{2} \times \frac{6}{5}$ means $\frac{1}{2}$ of $\frac{6}{5}$, which we know is $\frac{3}{5}$. Why? $\frac{2}{3}$ of 15 equals $2 \times \frac{1}{3}$ of 15, or 2×5 which is 10. It is generally simpler to use the following form:

$$\frac{2}{3} \times \frac{5}{15} = 2 \times 5 = 10; \quad \frac{3}{4} \times \frac{8}{21} = \frac{2}{7}.$$

Multiplication of several fractions is in general carried out in this manner. Thus:

$$\frac{2}{3} \times \frac{7}{16} \times \frac{12}{35} = \frac{1}{10}.$$

$$\begin{matrix} 4 \\ 4 \\ 4 \\ 5 \\ 2 \end{matrix}$$

EXERCISES

1. Explain the cancellation in each example above.
Use pencil and paper only when absolutely necessary in carrying out the following:

- | | | |
|--------------------------------------|---|---|
| 2. $6 \times \frac{2}{3}$ | 11. $\frac{3}{8}$ of $\frac{4}{15}$ | 20. $\frac{2}{3} \times \frac{15}{16} \times \frac{4}{5}$ |
| 3. $7 \times \frac{5}{21}$ | 12. $\frac{2}{5}$ of $\frac{15}{16}$ | 21. $\frac{3}{4}$ of $\frac{4}{5}$ of 20 |
| 4. $\frac{3}{4} \times \frac{8}{15}$ | 13. $\frac{5}{6}$ of $\frac{12}{25}$ | 22. $\frac{2}{3}$ of $6 \times \frac{3}{4}$ |
| 5. $\frac{1}{4} \times \frac{9}{5}$ | 14. $\frac{9}{10}$ of $\frac{15}{18}$ | 23. $\frac{5}{6}$ of $\frac{2}{3}$ of 60 |
| 6. $8 \times \frac{5}{24}$ | 15. $\frac{8}{10} \times \frac{25}{36}$ | 24. $\frac{5}{9}$ of $\frac{3}{4} \times \frac{24}{35}$ |
| 7. $10 \times \frac{3}{4}$ | 16. $\frac{5}{8}$ of $\frac{12}{25}$ | 25. $\frac{5}{6} \times 18 \times \frac{7}{10}$ |
| 8. $\frac{4}{15}$ of 12 | 17. $\frac{8}{15} \times \frac{25}{32}$ | 26. $\frac{15}{16} \times 12 \times \frac{7}{10}$ |
| 9. $\frac{2}{3}$ of 12 | 18. $\frac{7}{8} \times \frac{2}{21}$ | 27. $45 \times \frac{7}{18} \times \frac{4}{5}$ |
| 10. $15 \times \frac{8}{35}$ | 19. $\frac{3}{10} \times \frac{8}{35}$ | 28. $\frac{7}{15} \times \frac{12}{35} \times \frac{5}{12}$ |

46. Multiplication of Mixed Numbers.—Small mixed numbers are multiplied together by first reducing them to improper fractions. Thus:

$$2\frac{1}{2} \times 1\frac{1}{3} = \frac{5}{2} \times \frac{6}{3} = 3.$$

$$\begin{array}{r} 42\frac{2}{3} \\ 36\frac{1}{3} \\ \hline \end{array}$$

$\frac{2}{3} \times \frac{2}{3}$

$$25\frac{1}{3} \dots \frac{2}{3} \times 42$$

$$24 \dots 36 \times \frac{2}{3}$$

$$\left. \begin{array}{r} 252 \\ 126 \\ \hline 1561\frac{2}{3} \end{array} \right\} 36 \times 42$$

Products of larger mixed numbers are found without this reduction by means of these five distinct operations.

EXERCISES

Find the following products in the simplest manner, using pencil and paper only when necessary:

- | | | |
|--|---|--|
| 1. $2\frac{1}{2} \times 1\frac{1}{4}$ | 8. $1\frac{1}{5} \times \frac{1}{6}$ | 15. $64\frac{2}{3} \times 75\frac{1}{6}$ |
| 2. $3\frac{1}{2} \times 4$ | 9. $\frac{4}{5} \times \frac{2}{7} \times 35$ | 16. $35 \times \frac{3}{4} \times 92\frac{3}{4}$ |
| 3. $4\frac{1}{2} \times 10\frac{1}{4}$ | 10. $3\frac{1}{5} \times 2\frac{1}{3}$ | 17. $48\frac{3}{5} \times 74\frac{5}{6}$ |
| 4. $5\frac{1}{2} \times 6$ | 11. $3\frac{1}{4} \times \frac{1}{3} \times 24$ | 18. $54\frac{2}{3} \times 25\frac{2}{3}$ |
| 5. $3\frac{1}{4} \times 8\frac{1}{5}$ | 12. $5\frac{1}{2} \times \frac{5}{22}$ | 19. $67\frac{2}{3} \times 65\frac{2}{3}$ |
| 6. $4\frac{1}{3} \times 1\frac{1}{5}$ | 13. $3\frac{1}{5} \times \frac{1}{8} \times 15$ | 20. $28\frac{2}{3} \times 79\frac{3}{4}$ |
| 7. $4\frac{3}{4} \times 8$ | 14. $5\frac{1}{6} \times 1\frac{1}{5}$ | 21. $17\frac{4}{5} \times 34\frac{1}{2}$ |

22. Mr. Smith drives 30 mi. per hour in his automobile. How far does he go in $4\frac{1}{2}$ hr. ? in $5\frac{1}{3}$ hr. ? in $8\frac{1}{4}$ hr. ?

23. A recipe calls for $4\frac{1}{2}$ cupfuls of flour. How much flour is needed to make $\frac{1}{2}$ the recipe? $1\frac{1}{2}$ the recipe?

24. Coal sells for $\$5\frac{3}{4}$ per ton. What is the cost of 3 T. ? of $2\frac{1}{2}$ T. ? of $4\frac{1}{2}$ T. ? of $8\frac{1}{2}$ T. ?

25. When eggs sell for 36¢ per dozen, what will be the price of $2\frac{1}{2}$ doz. ? of $3\frac{1}{4}$ doz. ? of $5\frac{2}{3}$ doz. ?

47. Division of Fractions by Whole Numbers.—Division is always the opposite of multiplication. $\frac{4}{5} \div 3$ indicates that a fraction is to be found which multiplied by 3 equals $\frac{4}{5}$. Since $3 \times \frac{4}{5} = \frac{12}{5}$, therefore $\frac{4}{5} \div 3 = \frac{4}{15}$. Whenever possible divide the numerator by the divisor. Thus:

$$\frac{8}{15} \div 4 = \frac{2}{15}; \quad \frac{15}{8} \div 5 = \frac{3}{16}.$$

It may not be possible to divide the numerator by the divisor, as $\frac{4}{5} \div 6$. We now seek a fraction which if multiplied by 6 gives $\frac{4}{5}$. The fraction to be found must then have 6 a factor of the denominator. Such would be $\frac{2}{5} \times \frac{1}{6}$, which reduces to $\frac{1}{15}$. Simply carry out the cancellation in $\frac{2}{5} \times \frac{1}{6}$.

EXERCISES

Solve as many of the following as possible without the use of pencil and paper:

1. $\frac{3}{7} \div 6$

7. $\frac{8}{9} \div 6$

13. $\frac{3}{4} \times 8$

2. $\frac{8}{9} \div 7$

8. $\frac{4}{7} \div 8$

14. $\frac{5}{6} \times 10$

3. $\frac{4}{5} \div 8$

9. $\frac{8}{27} \div 4$

15. $\frac{3}{4} + \frac{2}{3} - \frac{5}{6}$

4. $\frac{2}{3} \div 4$

10. $\frac{8}{27} \div 27$

16. $\frac{4}{5} \times \frac{7}{8} \times 10$

5. $\frac{2}{3} \div 3$

11. $\frac{7}{8} \div 8$

17. $\frac{7}{12} \div 14$

6. $\frac{8}{17} \div 4$

12. $\frac{4}{5} \div 12$

18. $\frac{7}{8} \div 21 \times 36$

19. The bricks and the mortar between them measure $8\frac{1}{2}$ in. for 3 bricks. What does one brick and one layer of mortar measure?



20. In a school of 36 pupils the teacher found that there had been 45 half-day absences in a school month of 20 da. What part of all the school time of all the pupils was this?

48. Division of Fractions by Fractions.—Division of one fraction by another must also be thought of as the opposite of multiplication. $\frac{2}{3} \div \frac{7}{5}$ equals the fraction which multiplied by $\frac{7}{5}$ gives $\frac{2}{3}$. Such a fraction is $\frac{2 \times 7}{3 \times 5}$ or $\frac{2}{3} \times \frac{7}{5}$, as there must be 2×7 in the numerator and 3×5 in the denominator to cancel the 5 and 7 in $\frac{7}{5}$ when multiplied by the last fraction and still leave $\frac{2}{3}$. Hence, to divide by a fraction, invert the divisor and multiply. Thus:

$$\frac{4}{5} \div \frac{6}{35} = \frac{4}{5} \times \frac{35}{6} = \frac{14}{3} = 4\frac{2}{3}$$

EXERCISES

- | | | |
|---|---|---|
| 1. $\frac{2}{3} \div \frac{5}{6}$ | 10. $\frac{4}{5} \times \frac{15}{8}$ | 19. $\frac{3}{4} \times \frac{5}{6} \div \frac{1}{6}$ |
| 2. $\frac{12}{5} \div 4$ | 11. $\frac{2}{7} \div \frac{8}{21}$ | 20. $\frac{7}{8} \div \frac{3}{4} \times \frac{6}{5}$ |
| 3. $\frac{1\frac{1}{2}}{2} \div \frac{3}{4}$ | 12. $\frac{1\frac{1}{2}}{2} \times \frac{8}{9}$ | 21. $\frac{8}{10} \div \frac{6}{7} \times 8$ |
| 4. $\frac{1\frac{2}{3}}{1\frac{1}{2}} \div 7$ | 13. $\frac{1\frac{1}{2}}{5} \div \frac{4}{5}$ | 22. $\frac{1\frac{1}{2}}{1} \div 6 \times 34$ |
| 5. $\frac{8}{11} \div \frac{25}{164}$ | 14. $\frac{27}{5} \div \frac{9}{8}$ | 23. $\frac{7}{8} \times \frac{4}{15} \times \frac{45}{8}$ |
| 6. $8 \div \frac{3}{4}$ | 15. $\frac{4}{5} \div 8$ | 24. $\frac{3}{5} \times \frac{2}{3} \div \frac{7}{9}$ |
| 7. $\frac{3}{5} \div \frac{1}{15}$ | 16. $\frac{4}{7} \times \frac{31}{2}$ | 25. $\frac{5}{8} \times \frac{6}{7} \times \frac{15}{28}$ |
| 8. $\frac{1\frac{1}{2}}{6} \div 9$ | 17. $\frac{5}{7} \div \frac{3}{10}$ | 26. $\frac{3}{4} \times \frac{1}{2} \div \frac{5}{6}$ |
| 9. $\frac{27}{8} \div \frac{9}{16}$ | 18. $\frac{63}{100} \div 9$ | 27. $\frac{2}{3} \times \frac{7}{8} \div \frac{14}{15}$ |

28. If $\frac{2}{3}$ of a number equals 24, what is the number?
29. What number increased by $\frac{1}{3}$ of itself equals 16?
30. A man invested $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{2}{5}$ of his money and had \$500 left. How much had he at first?
31. A baseball team won $\frac{3}{5}$ of the games it played in a certain season. If it lost 18 games, how many did it win?
32. Play a number game, using multiplication and division of fractions.

49. Division of Mixed Numbers.—Division of a mixed number by a fraction may be performed by simply multiplying the dividend by the divisor inverted. Thus:

$$34\frac{2}{3} \div \frac{2}{5} = 34\frac{2}{3} \times \frac{5}{2} = 86\frac{1}{3}.$$

Division of a mixed number by another mixed number can be performed by reducing the divisor to an improper fraction, inverting this, and multiplying. Thus:

$$24\frac{1}{2} \div 2\frac{1}{3} = 24\frac{1}{2} \div \frac{7}{3} = 24\frac{1}{2} \times \frac{3}{7} = ?$$

Division of a mixed number by a whole number may often be shortened by dividing the whole number of the dividend by the divisor first. Then add any remainder to the fraction in the dividend. Divide this fraction by the divisor. Thus:

$$45\frac{2}{3} \div 4 = 11 + (1 + \frac{2}{3}) \div 4 = 11 + \frac{5}{3} \div 4 = \\ 11 + \frac{5}{12} = 11\frac{5}{12}.$$

EXERCISES

Carry out the following operations:

- | | | |
|---|--|---|
| 1. $3\frac{1}{4} \div \frac{2}{3}$ | 8. $1\frac{2}{3} \div 3\frac{1}{4}$ | 15. $\frac{2}{3} \div \frac{4}{5} \div \frac{3}{4}$ |
| 2. $4\frac{2}{5} \div \frac{1}{15}$ | 9. $8\frac{2}{3} \div 4\frac{1}{6}$ | 16. $25\frac{2}{3} \times 12\frac{1}{4}$ |
| 3. $\frac{8}{5}\frac{6}{7} \div 7$ | 10. $5\frac{3}{4} \div 4\frac{7}{8}$ | 17. $35\frac{3}{4} \div 7$ |
| 4. $4\frac{2}{5} \div 1\frac{1}{5}$ | 11. $2\frac{3}{5} \div \frac{2}{3}\frac{6}{7}$ | 18. $29\frac{1}{3} \div 4$ |
| 5. $\frac{2}{3}\frac{5}{7} \times 1\frac{8}{5}$ | 12. $8\frac{1}{3} \div 4$ | 19. $5\frac{1}{4} \div 1\frac{7}{20}$ |
| 6. $2\frac{1}{2} \div 3\frac{1}{4}$ | 13. $3\frac{2}{5} \times 15\frac{1}{4}$ | 20. $25\frac{2}{3} \div 7$ |
| 7. $6\frac{1}{2} \div 3\frac{1}{4}$ | 14. $7\frac{2}{3} \div 3$ | 21. $8\frac{2}{3} \div 2\frac{3}{5}$ |

22. What is the cost of 254 bu. wheat at \$ $1\frac{3}{5}$ per bushel?
23. A merchant bought cloth at \$ $2\frac{1}{5}$ per yard. For what must he sell it to make $\frac{1}{5}$?
24. If there are 36 in. in a yard, how many yards are there in 72 in.? in 42 in.? in 56 in.? in 48 in.?
25. How many pieces $\frac{2}{3}$ yd. wide can be made from a piece of cloth $8\frac{3}{4}$ yd. long?

50. Fractions a Form of Division.—One number divided by a second may be expressed as a fraction. The dividend is made the numerator and the divisor the denominator of the fraction. $12 \div 3$ and $1\frac{2}{3}$ will each reduce to 4. In all other divisions the division and the reduction of the fraction made as suggested will be the same. Try three other divisions.

51. Complex Fractions.—Fractions often arise in which either or both terms are themselves fractions or mixed numbers. Thus:

$$\frac{\frac{3}{4}}{5}, \quad \frac{5\frac{2}{3}}{\frac{2}{5}}, \quad \frac{4\frac{1}{2}}{5\frac{2}{3}}, \text{ etc.}$$

Such fractions are called **complex fractions**.

To simplify a complex fraction means to change it to a fraction or a mixed number. As a fraction is a form of division, a complex fraction can be written as the numerator divided by the denominator. Thus:

$$\frac{\frac{5\frac{1}{3}}{6\frac{1}{3}}}{=} = 5\frac{1}{3} \div 6\frac{1}{3} = 5\frac{1}{3} \div \frac{55}{9} = ?$$

EXERCISES

1. Write six complex fractions.
2. Simplify these six complex fractions.

Simplify the following:

3. $\frac{5\frac{5}{6}}{\frac{7}{8}}$

6. $\frac{1\frac{3}{4} + 3\frac{1}{2}}{4\frac{3}{8}}$

9. $\frac{5 - 3\frac{1}{2}}{2 - \frac{1}{8}}$

4. $\frac{4\frac{2}{5}}{1\frac{1}{5}}$

7. $\frac{\frac{1}{2} + \frac{2}{3}}{2\frac{1}{2}}$

10. $\frac{5 - 3\frac{3}{4}}{6 + 1\frac{1}{2}}$

5. $\frac{1\frac{2}{5}}{3\frac{3}{5}}$

8. $\frac{\frac{3}{4} + \frac{5}{6}}{1 - \frac{1}{2}\frac{1}{7}}$

11. $\frac{\frac{3}{4} + \frac{5}{6}}{\frac{9}{14} - \frac{8}{15}}$

12. Play a number game, using examples in fractions.

52. Aliquot Parts.—Do you use the simple forms for multiplication given in Arts. 25, 26, 27? The following will also be found useful:

$$12\frac{1}{2} = \frac{1}{8} \text{ of } 100$$

$$33\frac{1}{3} = \frac{1}{3} \text{ of } 100$$

$$16\frac{2}{3} = \frac{1}{6} \text{ of } 100$$

$$66\frac{2}{3} = \frac{2}{3} \text{ of } 100$$

$$75 = \frac{3}{4} \text{ of } 100$$

To multiply by $12\frac{1}{2}$ multiply by 100 and divide by 8.
Why? Find $324 \times 12\frac{1}{2}$.

These short cuts can be used in finding such products as $324 \times 512\frac{1}{2}$.

$$\begin{array}{r} 324 \\ \times 512\frac{1}{2} \\ \hline 4050 & 12\frac{1}{2} \times 324 \text{ which is } \frac{1}{8} \text{ of } 32,400. \\ 162000 & 500 \times 324 \\ \hline 166050 \end{array}$$

Use short cuts whenever they arise; therefore learn their use thoroughly.

EXERCISES

Use pencil and paper in the following only when you cannot find the result any other way:

- | | | |
|-------------------------------|-----------------------------------|--|
| 1. $432 \times 12\frac{1}{2}$ | 9. $432 \times 312\frac{1}{2}$ | 17. $1\frac{5}{8} \div 5$ |
| 2. $642 \times 33\frac{1}{3}$ | 10. 739×225 | 18. $1\frac{8}{15} \div 1\frac{2}{5}$ |
| 3. $1,357 \times 25$ | 11. $345 \times 233\frac{1}{3}$ | 19. $7,357 \times 325$ |
| 4. 716×75 | 12. $74,573 \times 50$ | 20. $\frac{2}{5} \text{ of } 3\frac{1}{8}$ |
| 5. $546 \times 16\frac{2}{3}$ | 13. $463 \times 533\frac{1}{3}$ | 21. $465 \times 266\frac{2}{3}$ |
| 6. 512×75 | 14. $615 \times 566\frac{2}{3}$ | 22. $726 \times 433\frac{1}{3}$ |
| 7. $726 \times 33\frac{1}{3}$ | 15. $9,327 \times 133\frac{1}{3}$ | 23. $1\frac{5}{8} \div 3\frac{3}{4}$ |
| 8. $25,367 \times 50$ | 16. $7,548 \times 325$ | 24. $3\frac{1}{8} \div 1\frac{7}{8}$ |

25. Play a number game, using examples in simplifying multiplication and division.

53. Approximations.—It is often desirable to know about how far it is to a certain city, about what an article will cost, or about how much time it will require to do a certain piece of work, rather than the exact distance, the exact cost, or the exact time. We should hence learn to make approximations, or estimates, accurately and quickly. For instance, $224 \times 19\frac{3}{4}$ will be very nearly 220×20 , or 4400.

What will $8\frac{1}{2}$ T. coal cost at $\$5\frac{3}{4}$ per ton?

This is nearly 8 T. at \$6 per ton, or \$48. The price will hence be nearly \$50.

Such approximations are also a help in checking computations. If the computed price for the above coal had been much above or below \$50 the result would surely have been incorrect. In all future problems make an estimate first and later compare the computed result with this estimate. Practise making these estimates to see how close you can come to the computed results.

EXERCISES

Approximate the results for the following problems which you have already solved and compare these with the computed results:

1. Page 14, Exs. 8, 9, and 10.
2. Page 21, at the bottom, Exs. 1, 2, and 3.
3. Page 23, at top, Exs. 2, 3, and 4.
4. Page 25, Exs. 1, 2, and 3.
5. Page 26, Exs. 9, 10, 11, 14, 15, and 16.
6. Page 34, Exs. 22, 23, and 25.
7. Page 36, Exs. 22, 23, 24, and 25.
8. Page 39, Exs. 22, 23, 24, and 25.
9. About how long has the Hindu number system been used? See page 4.
10. Play a number game, using examples in fractions and approximations.

54. Solution of Problems.—First reread Arts. 14 and 31.

Find the cost of 450 lb. of oats at 56¢ per bushel of 32 lb.

In the above problem the total price is wanted. As the price of one bushel is given, the total price is obtained by multiplying the number of bushels by the price of one bushel. The number of bushels in 450 lb. is found by dividing 450 by 32, as there are 32 lb. in each bushel. In the place of dividing 450 by 32 merely indicate it; thus, $\frac{450}{32}$. Then indicate that this—the number of bushels—is multiplied by 56. Thus:

$$\frac{450}{32} \times 56.$$

The work may now be greatly shortened by cancellation.

$$\begin{array}{r} 450 \\ \hline 32 \\ 4 \end{array} \quad \begin{array}{r} 7 \\ \times 56 = ? \end{array}$$

Put all of your attention on:

1. *Reading and understanding the problem;*
2. *Putting down the numbers to indicate the necessary operations;*
3. *The numerical work needed—including checks;*
4. *Interpreting the result, which here is in dollars or cents.*

By following these suggestions you can keep your attention upon one thing at a time and save work in computations.

Be constantly on the lookout for an opportunity to use the opposite processes, cancellation, or any other short cut in computations. Check all final results. Before finding the exact result always estimate it as accurately as possible.

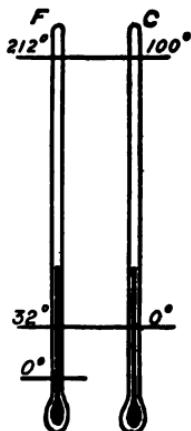
EXERCISES

1. Explain the four processes used in solving problems. Illustrate by solving problems 3 and 4 which follow.

2. Show the advantage of indicating the number work first before doing any of it.

3. What is the cost of 1,260 lb. of oats at 60¢ per bushel?

4. A dealer sells goods that are out of season at $\frac{1}{4}$ below the marked price. For what will he sell an article marked at \$12? at \$6 $\frac{1}{2}$? at \$3 $\frac{1}{4}$? at \$18? at \$4 $\frac{1}{2}$?



5. Oilcloth 30 in. wide costs 28¢ per yard. At the same rate what will be the cost of oilcloth 42 in. wide? 36 in. wide?

6. Five degrees on the Centigrade thermometer equal 9 degrees on the Fahrenheit thermometer. What will be the change in the Fahrenheit thermometer when the Centigrade falls 10 degrees? rises 12 degrees? falls 32 degrees? falls 22 $\frac{1}{2}$ degrees?

7. What will be the change in the Centigrade thermometer when the Fahrenheit falls 36 degrees? rises 13 $\frac{1}{2}$ degrees? rises 10 $\frac{1}{2}$ degrees? falls 15 $\frac{2}{3}$ degrees?

8. Coal which sold for \$7 $\frac{1}{2}$ per ton has advanced $\frac{1}{10}$ in price. For what does it now sell?

9. For what fractional part of last year's price does coal now sell which has advanced $\frac{1}{10}$ over last year?

10. Coal which has advanced $\frac{1}{8}$ over last year sells for \$4 $\frac{1}{2}$ per ton. What was the price last year? Last year's price was what fractional part of this year's price?

11. If coal which sold for \$5 $\frac{1}{2}$ per ton last year sells for \$6 $\frac{1}{2}$ per ton this year, what part of last year's price has it advanced? How much more will be the cost of 8 T.?

12. A farmer owning 160 acres of land sells $\frac{1}{2}$ and then $\frac{1}{2}$ of the remainder. What part of the land has he left? How many acres has he left? The acres he has left is what fraction of the land he sold?

Often a fraction has no meaning. If such a fraction is $\frac{1}{2}$ or more, the number is made one larger; if the fraction is less than $\frac{1}{2}$ the fraction is merely omitted. Thus, $37\frac{1}{2}$ cents become 38 cents; $29\frac{3}{4}$ cents become 30 cents; $48\frac{1}{4}$ cents become 48 cents. Similarly, $7\frac{1}{2}$ footballs would be 7 footballs, $9\frac{5}{8}$ boys would be 10 boys. Give other illustrations.

13. Only $\frac{1}{20}$ of the business men of the United States are considered successful. At that rate how many are considered successful out of 420? out of 356? out of 164?

14. At the rate of 3 boxes of berries for 25¢, what will be the cost of a case of 12 boxes? a case of 15 boxes? a case of 32 boxes?

15. If John makes 6 safe hits in 25 times at bat, how many safe hits will he make at the same rate in 35 times at bat? in 56 times? in 100 times? in 47 times?

16. If Henry makes 4 errors in 71 chances he has had in the field, how many errors would he make at the same rate in 36 chances? in 108 chances? in 15 chances?

17. How many correct plays would Henry make at the above rate in 42 chances? in 178 chances? in 28 chances?

18. A certain excursion fare is \$ $6\frac{7}{10}$. What will be the cost of 12 full and 9 half-fare tickets?

19. A merchant sold a suit of clothes for \$30 upon which he made $\frac{1}{5}$. What did the merchant pay for the suit?

20. Play a number game, using examples in fractions; also problems in fractions.

III

DECIMAL FRACTIONS

55. Meaning of Decimals.—How many are represented by each digit in 345? in 5,555? in 654? How is the value of a digit changed by moving the digit to the right? to the left?

About three hundred years ago mathematicians began to write fractions in a way similar to that in which whole numbers are written. Such fractions are called **decimal fractions**, or merely **decimals**. A period, called the **decimal point**, is placed before all such fractions. The first digit to the right of the decimal point takes $\frac{1}{10}$ the value it would in unit's place, the second digit takes $\frac{1}{100}$ the value it would in unit's place, and so on. Thus in 0.76 there are $\frac{7}{10}$ and $\frac{6}{100}$, which become $\frac{76}{100}$. How? Similarly, 0.045 is $\frac{4}{100}$ and $\frac{5}{1000}$, or $\frac{45}{1000}$. How?

In mixed numbers the whole number is separated from the fraction by the decimal point. Thus, 53.67, 4.065, and 845.0405.

56. Reading Decimals.—Decimals are read by first stating the whole number, and for the decimal point, and finally the decimal part. Thus, 245.617 is read "two hundred forty-five and six hundred seventeen thousandths."

57. Decimal Places.—The digits and zeros in a decimal are called its **decimal places**. How many decimal places are there in each number of Ex. 1, at the top of page 47?

EXERCISES

1. Read the following decimals: 0.45, 0.375, 4.25, 3.05, 45.67, 456.305, 30.00546, 304.40507, 4.00035, 0.00405.
2. Write the following decimals: twenty-five and sixty-nine hundredths; one hundred twenty-six and four hundred nine thousandths; ninety-seven and thirty-two tens of thousandths; five and two hundred twelve millionths.
3. Write and read the following as decimals: $\frac{15}{100}$, $\frac{175}{1000}$, $\frac{105}{100000}$, $\frac{845}{1000}$, $\frac{849}{1000000}$, $\frac{999}{1000}$.

58. Reducing Decimals to Common Fractions.—A decimal can easily be changed to a common fraction by writing the number over 10 multiplied by itself, so as to give as many zeros in the denominator as there are decimal places. Thus:

$$0.45 = \frac{45}{100} = \frac{9}{20}; \quad 3.04 = \frac{304}{100} = 3\frac{1}{25}.$$

59. Reducing Common Fractions to Decimals.—Common fractions whose denominators can be changed to 10, 100, etc., are easily converted into decimals. Thus:

$$\frac{3}{4} = \frac{75}{100} = 0.75; \quad \frac{8}{5} = 1\frac{3}{5} = 1\frac{6}{10} = 1.6.$$

In changing common fractions to decimals and vice versa only the form of the fraction is changed. Decimals are fractions in a different form. A little later we shall study yet another form of fractions.

EXERCISES

1. Change to decimals: $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{3}{5}$, $\frac{17}{10}$, $\frac{9}{4}$, $23\frac{1}{4}$, $6\frac{2}{3}$, $29\frac{1}{2}$, $715\frac{1}{4}$.
2. Change to common fractions or mixed numbers: 0.6, 0.45, 4.56, 23.16, 17.045, 0.0065, 143.00504, 60.35, 4.4056.
3. Which is larger, $\frac{2}{3}$ or 0.3? $\frac{3}{4}$ or 0.77? $\frac{5}{6}$ or 0.9?
4. Replace the common fractions on pages 44 and 45 by decimals and then read the problems.
5. Give the results in decimals for Exs. 1, 2, 3, 4, on page 28.

60. Reading Decimals.—How do business and scientific men read large numbers as 5,638 or 4,289? Similarly, they read 4,562.3107 “forty-five, sixty-two, point, thirty-one, oh, seven”; 63405.6029 “six thirty-four, oh, five, point, sixty, twenty-nine.”

EXERCISES

Write the following numbers in words: 265.32; 3,256; 8.56; 45.603; 4.5608; 23.0045; 35.6718.

61. Addition and Subtraction.—Decimals are added and subtracted in the same manner as whole numbers. This results from the fact that 10 units make 1 ten and 10 tens make 1 hundred; similarly, 10 hundredths make 1 tenth, 10 tenths make 1 unit. In writing numbers in a column to add or subtract, where must the decimal points come?

EXERCISES

Read the following numbers as a business or scientific man would. Add each column and check:

1. 4563.451	2. 5607.861	3. 7684.07	4. 5473.805
637.872	2463.754	8509.37	686.457
7468.930	3589.697	3806.45	7407.004
4575.340	793.005	658.37	6458.130
3607.872	7238.147	4616.86	8463.694
6218.459	6348.731	5875.34	5957.541
5007.735	4892.040	3782.41	3754.054
1469.452	3756.154	2301.37	2885.416
6458.341	8357.634	5607.74	7356.810
7349.742	5617.478	7318.92	6571.467
398.604	7894.072	1945.60	270.798
4856.793	5986.957	3877.93	1896.097
<u>3607.486</u>	<u>648.095</u>	<u>859.60</u>	<u>7358.186</u>

5. Where do columns of numbers to be added like the above arise? If possible obtain some and bring them to class to be added.

EXERCISES

Find the following differences and check results:

1. 375.76	4. 4235.604	7. 4506.325	10. 5672.036
<u>147.34</u>	<u>2568.42</u>	<u>2738.247</u>	<u>2984.153</u>
2. 74.607	5. 6325.41	8. 6231.	11. 5453.32
<u>35.439</u>	<u>1453.725</u>	<u>3219.127</u>	<u>2391.431</u>
3. 5703.605	6. 3657.291	9. 3507.206	12. 3672.194
<u>2436.827</u>	<u>2160.345</u>	<u>2149.13</u>	<u>2984.236</u>

13. Play a number game, using decimals.

62. Multiplication.—The product of $\frac{7}{10}$ and $\frac{23}{100}$ is $\frac{161}{1000}$. This reduces to 0.0161. Hence, the product of 0.7 and 0.023 can be found by multiplying the numbers together and giving this product as many decimal places as those of the multiplicand and multiplier added together. This rule extends to mixed numbers.

Multiplication and division of decimals may be checked by casting out the nines just as in whole numbers. Note that this does not check the position of the decimal point.

EXERCISES

Read the following as a business man or a scientist would and find the products. Check results.

1. 4653.132	4. 467.045	7. 305.467	10. 483.508
<u>24</u>	<u>5.734</u>	<u>36.746</u>	<u>50.734</u>
2. 4562.314	5. 708.36	8. 7468.46	11. 768.407
<u>13.5</u>	<u>74.831</u>	<u>7.08</u>	<u>5.708</u>
3. 4173.316	6. 567.405	9. 5709.451	12. 6738.905
<u>34.72</u>	<u>73.85</u>	<u>36.706</u>	<u>45.76</u>

63. Abbreviated Multiplication.—In finding the price of 14.97 T. of coal at \$12.35, only the last two decimal places are used. Why? Instead of carrying out the multiplication in full and dropping the useless decimal places, the work can be abbreviated and yet be fully as accurate.

- | | |
|--|---|
| $\begin{array}{r} \$12.34 \\ + 1.497 \\ \hline 14.97 \end{array}$
$\begin{array}{r} 49.36 \quad (1) \\ + 123.40 \quad (2) \\ \hline 172.76 \quad (3) \\ + .86 \quad (4) \\ \hline 184.73 \quad (5) \end{array}$ | <ol style="list-style-type: none"> 1. Multiplying by 4 gives 49.36. 2. Multiplying by 10 gives 123.40 3. Multiplying by .9 would give 3 decimal places.
$.9 \times .04$ gives .036. Instead of carrying 3 we carry 4. If the third decimal had been 4 or less, we should have carried only 3. This gives 11.11. A line is drawn through 4 to show that it will not enter into the next multiplication. 4. Multiplying 3 by .07—since 4 is omitted—gives .021, or 2 to carry. Hence, multiplication by .07 gives .86. 5. Adding the numbers as usual gives 184.73. |
|--|---|

EXERCISES

Read the following as a business or scientific man would and find the products to two decimal places:

- | | |
|---------------------------|----------------------------|
| 1. $4,563.37 \times 5.6$ | 7. $8,405.26 \times 5.64$ |
| 2. $3,207.26 \times 3.8$ | 8. $7,508.94 \times 4.6$ |
| 3. $6,456.14 \times 6.8$ | 9. $3,782.14 \times 20.6$ |
| 4. $4,305.76 \times 8.7$ | 10. $2,035.6 \times 5.68$ |
| 5. $6,074.35 \times 27.5$ | 11. $3,200.84 \times 4.63$ |
| 6. $7,236.75 \times 4.76$ | 12. $7,463.06 \times 5.08$ |

Carry out the following to three decimal places:

- | | |
|---------------------------|----------------------------|
| 13. 43.65×3.1416 | 18. 94.805×7.08 |
| 14. 9.607×4.056 | 19. 376.809×5.87 |
| 15. 745.06×8.453 | 20. 305.761×0.706 |
| 16. 35.056×4.67 | 21. 46.748×7.056 |
| 17. 73.405×0.045 | 22. 38.608×5.406 |
23. Play a number game, using abbreviated multiplication of decimals.

64. Division.—Division of a decimal by a whole number is carried out quite similarly to that of division of a whole number by another whole number.

$\begin{array}{r} 17.433 \\ \hline 324) 5648.374 \end{array}$	324 goes into 546 tens, 1 ten times.
$\begin{array}{r} 324 \\ \hline 2408 \end{array}$	324 goes into 2,408 units, 7 units times.
$\begin{array}{r} 2268 \\ \hline 1403 \end{array}$	324 goes into 1,403 tenths, 4 tenths times.
$\begin{array}{r} 1296 \\ \hline 1077 \end{array}$	324 goes into 1,077 hundredths, 3 hundredths times.
$\begin{array}{r} 972 \\ \hline 1054 \end{array}$	324 goes into 1,054 thousandths, 3 thousandths times.
$\begin{array}{r} 972 \\ \hline 82 \end{array}$	82 Remainder in thousandths.

If the divisor contains a decimal, multiply divisor and dividend each by 10 as many times as is necessary to make the divisor a whole number. This gives the same kind of division as the above. $24.357 \div 3.72 = 2,435.7 \div 372$.

EXERCISES

Read the following, carry out the divisions, and check:

- | | |
|---------------------------|--------------------------|
| 1. $45.386 \div 4.35$ | 7. $84.5607 \div 25.04$ |
| 2. $356.6407 \div 34.5$ | 8. $37.8205 \div 46.8$ |
| 3. $53.4078 \div 56.73$ | 9. $50.417 \div 4.63$ |
| 4. $84.3607 \div 29.5$ | 10. $46.4075 \div 24.7$ |
| 5. $3,462.5074 \div 25.8$ | 11. $4.56073 \div 64.8$ |
| 6. $475.386 \div 18.45$ | 12. $56.0457 \div 35.14$ |

13. William works for \$7.50 per week. How much is this per day?

14. Mr. Grey bought a lot 66 ft. front for \$650. What is this per front foot?

65. Abbreviated Division.—Division of decimals can also be abbreviated. Divide 173,488 by 4,892 to 3 decimal places.

$$\begin{array}{r} 35.464 \\ \underline{4892)173488} \\ 14676 \quad 1) \\ \underline{26728} \\ 24460 \quad 2) \\ \underline{2268} \\ 1957 \quad 3) \\ \underline{311} \\ 293 \quad 4) \\ \underline{18} \\ 19 \quad 5) \end{array}$$

- ✓
1. $17,348$ tens $\div 4,892$ gives 3 tens.
 2. $26,738$ units $\div 4,892$ gives 5 units.
 3. Instead of annexing zeros to the dividend cut off the last digit from the divisor and divide $2,268$ tenths by 489 giving 4 tenths. In multiplying 489 by 4, note that the 2, cut off, times 4 gives 8. Hence 1 is carried to 4×9 , which is then $36 + 1$, or 37.
 4. Next cut off 9 from the divisor and divide 311 hundredths by 48 , giving 6 hundredths. In multiplying by 6, note that $6 \times 9 = 54$, of that 5 is carried to 6×8 , which is then $48 + 5 = 53$.
 5. Finally cut off 8 from the divisor and divide 18 thousandths by 4. This is 4 thousandths, since 4×8 gives 3 to carry to 4×4 .

The divisor must contain one digit more than the number of decimal places needed. A zero must be annexed to the dividend for each such digit lacking in the divisor. In $635 \div 19$ to 2 decimal places one zero must be annexed; in $56 \div 7$ to 2 decimal places two zeros must be annexed. Carry out these divisions.

EXERCISES

Find the following quotients by abbreviated division to 3 decimal places and check your results:

- | | |
|-----------------------|---------------------------|
| 1. $5,463 \div 4,562$ | 7. $1,356 \div 2,304$ |
| 2. $3,405 \div 2,405$ | 8. $459.5 \div 23.4$ |
| 3. $4,517 \div 3,085$ | 9. $42,561 \div 54$ |
| 4. $24.63 \div 34.15$ | 10. $7,304 \div 34$ |
| 5. $4,563 \div 156$ | 11. $4,516 \div 304$ |
| 6. $7,503 \div 628$ | 12. $0.5463 \div 0.01306$ |

13. Twenty-four dollars were paid for a railroad ticket to travel 723 mi. Find the cost per mile to three decimal places.

66. Reducing Common Fractions to Decimals.—Any fraction can be reduced to a decimal by merely dividing the numerator by the denominator, as $\frac{23}{42}$ means $23 \div 42$. Thus:

$$\frac{23}{42} = 0.5476190476190 \dots$$

67. Repeating Decimals.—In the above decimal, 476190 keeps on repeating. Such a decimal is called a **repeating decimal**, a **recurring decimal**, or a **circulating decimal**. They are written by placing a dot above the first and last digits of the repeating part. Thus, 0.5476190.

68. Reducing Repeating Decimals to Common Fractions.—It is shown in algebra that a repeating decimal can be reduced to a common fraction by the following rule: Write a fraction with the repeating part as the numerator. Write as many 9's in the denominator as there are digits in the numerator and annex zeros, if necessary, so that with the 9's they equal the number of decimal places of the decimal. Thus:

$$0.0\dot{4}5\dot{6} = \frac{456}{9990} = ? \text{ and } 0.4\dot{3}6\dot{5} = \frac{4}{10} + \frac{365}{9990} = ?$$

EXERCISES

1. Reduce to decimals, marking the repeating part: $\frac{2}{7}, \frac{3}{7}, \frac{1}{11}, \frac{1}{12}, \frac{2}{11}, \frac{1}{15}, \frac{1}{18}, \frac{1}{19}, \frac{2}{7}, \frac{3}{11}, \frac{1}{12}, \frac{1}{15}, \frac{1}{18}$.
2. Change to common fractions: 0.425, 0.0264, 4.136, 0.4635, 0.04562, 0.00456, 0.6, 0.045624, 5.4506.
3. The readings on a certain barometer are in inches and fractions of an inch. To compare the readings with the weather maps, the fractions must be changed to decimals. Give the following readings in decimals, without using pencil and paper: $29\frac{1}{4}$ in.; $28\frac{3}{4}$ in.; $29\frac{1}{2}$ in.; $28\frac{7}{8}$ in.; $29\frac{5}{8}$ in.

69. Aliquot Parts.—Show how to simplify multiplication by 5, $12\frac{1}{2}$, $16\frac{4}{5}$, 25, $33\frac{1}{3}$, 50, $66\frac{2}{3}$, and 75. Have you made use of these simplifications whenever possible?

Divide 3,245 and 567.8435 each by 10, by 100, and by 1,000. To divide by 200, 30, 4,000, etc., first divide by 2, 3, 4, etc., and then by 100, 10, 1,000, etc. In order to divide by any of the above aliquot parts the process is just the opposite of multiplying by them. To divide by 25, multiply by 4 and divide by 100. Thus, $635 \div 25 = 635 \times 4 \div 100 = 2,540 \div 100 = 25.4$.

EXERCISES

1. Explain how to multiply by the above aliquot parts.
2. Explain how to divide by the above aliquot parts.

Carry out the following operations in the simplest manner:

- | | | |
|--------------------------------|---------------------------------|---------------------------------|
| 3. 348×25 | 13. $645 \div 16\frac{2}{3}$ | 23. $3,564.23 \times 500$ |
| 4. $456 \div 25$ | 14. $6486 \div 500$ | 24. $7,364 \div 25$ |
| 5. $324 \times 16\frac{2}{3}$ | 15. $654 \div 50$ | 25. $5 \times 37 \times 2$ |
| 6. $213 \div 5$ | 16. $2 \times 719 \times 5$ | 26. $720 \times 12\frac{1}{2}$ |
| 7. 25×512 | 17. $642 \times 516\frac{2}{3}$ | 27. $25 \times 729 \times 4$ |
| 8. $48.6 \div 25$ | 18. 5.48×50 | 28. $22\frac{1}{2} \times 40$ |
| 9. $42 \times 12\frac{1}{2}$ | 19. $4.56 \div 75$ | 29. $32 \times 12\frac{1}{2}$ |
| 10. $926 \times 16\frac{2}{3}$ | 20. $372 \times 33\frac{1}{3}$ | 30. $50 \times 737 \times 2$ |
| 11. $824 \times 12\frac{1}{2}$ | 21. $732 \times 66\frac{2}{3}$ | 31. $346.28 \div 25$ |
| 12. 64.28×25 | 22. $348 \div 4.300$ | 32. $53.427 \div 33\frac{1}{3}$ |

33. There are 2,000 lb. in a ton. Without the use of pencil and paper express the following in tons and a decimal of a ton: 64,380 lb.; 24,340 lb.; 5,960 lb.; 4,370 lb.

34. At \$1.12 $\frac{1}{2}$ find the cost of 32 boxes of grapes; of 23 boxes; of 43 boxes.

35. How many yards of cloth at 25¢ per yard can be bought for \$12? for \$22? for \$56? for \$75?

36. Play a number game, using changes of fractions to decimals and vice versa; also aliquot parts.

70. Problems.—Reread page 43 very carefully. Be constantly on the lookout to use the opposite process in problems which are difficult for you. Estimate each result first and compare the final result with the estimate. Make use of aliquot parts, approximations, and all other processes which simplify computations.

EXERCISES

1. Change the fractions in Exs. 6 and 7, page 44, to decimals and solve the problems.
2. Change the fractions to decimals in Exs. 8, 10, and 11, on page 44, and solve the problems.
3. Change the fractions to decimals in Exs. 12 and 13, page 45.
4. Change the fractions to decimals in Exs. 18 and 19, on page 45.
5. Find the price of 6,840 lb. of coal at \$5.25 per ton.
6. Mr. Jones bought 13.583 T. of coal at one time, 4.62 T. at a second time, and 8.345 T. at a third. He paid \$158.30. What was the average price per ton?
7. An automobile cyclometer registered 2,342.8 mi. at the beginning of a trip and 2,578.3 mi. at its conclusion. What was the average rate per hour travelled if $8\frac{1}{4}$ hr. were consumed in making the trip?
8. The meter is 39.37 in. long. Find the length in inches of 4.25 meters; of 7.52 meters; of $5\frac{3}{4}$ meters.
9. How many meters are there in 3,425 in.? in 673.5 in.? in 56.48 in.? in 38.26 in.?
10. One inch equals what decimal part of a meter?



11. Of the 2,816,289 inhabitants of Georgia in 1917 it is estimated that 2,255,000 were farmers. How many out of every 1,000 were farmers? (Find to 3 decimal places what decimal part were farmers and multiply by 1,000.)
12. In 1817 the Georgia farmers raised crops valued at \$308,342,610. What was the average for each?

John's football team has won 5 and lost 3 games.

Express the following as common fractions and as 3-place decimals:

13. The number won is what part of the total number played?
14. The number lost is what part of the total number played?
15. The number lost is what part of the number won?
16. The number won is what part of the number lost?
17. At the above rate, how many games must the team play to win 8 games? to win 6? to win 12? to win 10?

A cubic foot of water weighs 62.5 lb. nearly. Ice is .92 as heavy as an equal volume of water.

18. What is the weight of 4.25 cu. ft. of water, to 2 decimal places?
19. Find to 2 decimal places the weight of 4 cu. ft. of ice; of 2.25 cu. ft. of ice; of 3.46 cu. ft. of ice.
20. Find to 2 decimal places how many cubic feet will be occupied by ice weighing 80 lb.; 24.36 lb.; 125.46 lb.; 1 T.
21. Find to 2 decimal places how many cubic feet will be occupied by water weighing 46 lb.; 8.25 lb.; 24.36 lb.
22. A cubic foot of water weighs how many times that of a cubic foot of ice?
23. Play a number game, using abbreviations in multiplication and division of decimals; also aliquot parts.

24. In a civil service examination 34 out of 168 applicants passed. Express by a 3-place decimal the part that passed.

25. At this rate how many would pass out of 315 ? 187 ? 369 ? 497 ?

26. One gallon contains 231 cu. in. How many gallons will a cubic foot of 1,728 cu. in. contain ?

27. How many gallons are there in 2.4 cu. ft. ? in 6.5 cu. ft. ? in 8.6 cu. ft. ?

28. One gallon is what part of a cubic foot ?

29. One bushel contains 2150.42 cu. in. How many gallons are there in a bushel ?

The following was the wheat yield in bushels for two different years, in the various American countries:

	1916-17	1915-16
United States.....	80,000,000	126,375,000
Canada.....	25,000,000	47,038,000
Argentina.....	9,600,000	21,000,000
Uruguay.....	600,000	1,375,000
Chile.....	2,500,000	2,650,000
Mexico.....	1,000,000	1,000,000

30. Find the total number of bushels produced in all America, for each of these two years.

31. Find to 3 decimal places, the part of the total produced by each country. Check by adding decimals. What should this sum be if the work has been carried out correctly ?

32. The total wheat crops for the entire world for these periods were 3,232,320,000 bu. and 4,171,784,000 bu. What part of the total wheat crop, was that of America for each of these years ?

33. How did the rate of decrease in wheat yield, for these years in the United States, compare with the decrease in all America and in the world ? Compute to 3 decimal places.

IV

PERCENTAGE

71. Meaning of Percentage.—Many hundred years ago, before simple decimals were invented, merchants found it convenient to count by hundreds. Their fractions all had the same denominator, 100. For our $\frac{1}{4}$ or .25 they used $\frac{25}{100}$. This form of counting has come down to us and we find the business men to-day changing fractions to those with a denominator 100 and speaking of the numerator as so many “per cent.” $\frac{1}{5}$ or .2, which reduces to $\frac{20}{100}$, we call 20 per cent. This is written 20 %. Any number of per cent means merely the numerator of a fraction whose denominator is 100. Thus, 13 per cent or 13 % is $\frac{13}{100}$, and 35 per cent or 35 % is $\frac{35}{100}$.

72. Reducing Per Cent to a Common Fraction or a Decimal.—Any per cent is changed to a common fraction or a decimal by first writing it as a common fraction with a denominator of 100—or merely thinking that this has been done—and simplifying. Thus:

$$60\% = \frac{60}{100} = \frac{3}{5}; 47.3\% = \frac{47.3}{100} = .473.$$

73. Reducing Common Fractions or Decimals to a Per Cent.—A common fraction or a decimal may be reduced to a per cent by reducing it to a common fraction with the denominator 100—or merely thinking that this has been done—when the numerator is the required per cent. Thus:

$$\frac{3}{4} = \frac{75}{100} = 75\%; .115 = \frac{11.5}{100} = 11.5\%.$$

EXERCISES

1. Replace the question-marks with the proper numbers:

$$\begin{array}{ll}
 \frac{4}{5} = . ? = ? \% & ? = .24 = ? \% \\
 ? = . ? = 15 \% & ? = .36 = ? \% \\
 \frac{7}{8} = . ? = ? \% & ? = . ? = 32 \% \\
 ? = .56 = ? \% & \frac{3}{4} = . ? = ? \% \\
 ? = . ? = 45 \% & \frac{7}{8} = . ? = ? \% \\
 \frac{11}{16} = . ? = ? \% & ? = . ? = 28 \% \\
 ? = .76 = ? \% & \frac{11}{16} = . ? = ? \% \\
 ? = . ? = 64 \% & ? = .72 = ? %
 \end{array}$$

2. Change the common fractions in Exs. 9 and 10, on page 44, to a per cent and read each problem as a problem in per cent.

3. Change the common fractions in Exs. 12 and 19, on page 45, to per cents and read each problem as a problem in per cent.

4. Change the following to per cents: $\frac{2}{3}$, $\frac{7}{25}$, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{8}{7}$, $\frac{5}{12}$, $\frac{7}{15}$, $\frac{3}{8}$, $\frac{1}{3}$, and $\frac{5}{8}$.

5. State Exs. 13, 14, 15, and 16, on page 56, as problems in per cent.

6. Change the following to per cents: .65, .04, 1.25, 3.06, 2.67, .08, .145, .325, .875, 1.045, 2.056, and .007.

7. Change the following to decimals: 45 %, 6 %, 245 %, .045 %, 345 %, 34.5 %, 4.5 %, 12.5 %, 16.5 %, 2.345 %, and 34.56 %.

8. A merchant invests \$1,500 so as to gain 15 % on the money invested. State this as a problem in decimals.

9. Which is the larger, $\frac{8}{25}$ or 13 %? $2\frac{1}{5}$ or 221 %? .345 or 35 %? 2.67 or 268 %? .125 or 12 $\frac{1}{2}$ %?

10. Play a number game, using changes as in the first exercise above.

74. Problems Containing Per Cent.—There is nothing new in problems containing a per cent which has not been learned in common fractions or in decimals. The only thing new is the language. In the place of the common fraction or the decimal the per cent is used. In the home common fractions are used mostly, as in $\frac{1}{4}$ of a pie. Scientists depend largely upon decimals, as in .25 of a foot. The business man uses percentage largely, as in decreasing expenses 25 %. The same fraction was used in each case, but it was stated in a different way. The following problems may be stated either in common fractions or in decimals:

A clerk's yearly salary of \$1,200 was raised 20 %. How many dollars was his salary raised?

A merchant marks his goods 20 % above what he paid for them. What should he mark an overcoat for which he paid \$23?

State each of these problems, using a common fraction in place of the per cent. State each problem, using a decimal in the place of the per cent.

Each per cent in a problem must first be reduced to a decimal before any operations are carried out. This changes every per cent problem to one in decimals.

EXERCISES

1. A merchant invests \$1,500 upon which he expects to gain 25 %. How much does he expect to gain?
2. A merchant invested \$3,500 so as to gain 12 %. What was his gain? (Find 12×35 mentally.)
3. How much is a loss of $33\frac{1}{3}\%$ on an investment of \$600? \$1,500? \$450?
4. Jane has spelled about 96 % of the words in her spelling-lessons correctly throughout the year. How many words should she spell correctly at that rate out of 300 words? out of 500? out of 324?

5. For six months Mary has averaged solving 90 % of the problems in her mathematics lesson. How many should she solve at that rate out of an assignment of 20 problems ? out of an assignment of 34 problems ?

6. James has solved on the average only $\frac{2}{3}$ as many problems as Mary. What per cent of the lesson assignment did James average ? How many would that be out of an assignment of 35 problems ?

7. In a certain locality in the United States corn loses 15 % of its weight from the time it is husked in the fall until spring. Corn which weighs 3,450 lb. when husked will lose how much weight by spring ?

8. 4.3 % of corn is fat, 7.8 % is protein, and 66.5 % is carbohydrate. Express each as a decimal.



9. Find the number of pounds of fat, protein, and carbohydrates in a ton of corn; in 350 lb.; in 10 lb.

Wheat flour contains 1.1 % fat — 11.3 % protein — 74.6 % carbohydrates

Meat	"	1.5 %	"	16.0 %	"	0.0 %	"
------	---	-------	---	--------	---	-------	---

Potatoes	"	.1 %	"	1.8 %	"	15.3 %	"
----------	---	------	---	-------	---	--------	---

Milk	"	4.0 %	"	3.3 %	"	5.0 %	"
------	---	-------	---	-------	---	-------	---

10. The remaining per cents are water and waste. Find the per cent of water and waste for each.

11. Make up problems for the class to solve from the above table. Try to make the best problems.

12. Which is more, 100 % of a number or the number itself ? Try your answer on several numbers.

13. Which is more, 200 % of a number or twice the number ?

14. Which is more, 325 % of a number or $3\frac{1}{2}$ times the number? Why is this so?
15. Increase 532 by 25 % of itself. Find 125 % of 532. Which is the larger?
16. Repeat Ex. 15, using a per cent other than 25 % and a number other than 532. What do you find? Whatever per cent or number is used the numbers found in this way will always be equal.
17. Mrs. Jackson bought a house for \$3,200 and sold it for 15 % above that price. How much did she gain? For how much did she sell the house?
18. William had a paper route of 65 subscribers. On Saturday he secured 20 % more. How many subscribers did he then have?
19. George has a paper route with 150 % more subscribers than William had at first. How many subscribers does George have?
20. John bought a paper route for \$65. After working it up so as to greatly increase the number of subscribers he sold it at an advance of 50 %. For how much did he sell the route? What was his gain for working up the route?
21. A man bought a house and sold it for 36 % above the purchase price. For what per cent of the purchase price did he sell the house?
22. Take 37 % of 350 away from itself. Subtract 37 % from 100 % and find this per cent of 350. Which is the larger of the two numbers?
23. Repeat Ex. 22 with other numbers than 350 and other per cents than 37 %. What do you find? Whatever the number and whatever the per cent used, the two numbers found in this way are equal.

24. A man bought an automobile for \$785. After using it five years he sold it for 45 % of what he paid for it. For how much did he sell the automobile? How much did he lose?

25. Doctor Percy bought an automobile for \$1,560. After using it for three years he sold it at a reduction of 65 % of the cost. For what per cent of the purchase price did he sell the automobile? How many dollars was this?

26. If corn loses 15 % of its weight from the time it is husked until spring, what per cent of the fall weight will it weigh in the spring? Corn that weighed 2,500 lb. in the fall will weigh what in the spring?

27. In our war with Germany 35 % of the men in the first draft were unfit for service. What per cent of the men called in this draft were fit for service?

28. How many is it expected would be fit out of a call of 2,000 men? 385 men? 1,250 men?

29. It is estimated that an average of 7.5 % of the food brought into the American home is wasted. What per cent is used?

30. In a home where the grocery bill averages \$45 per month, how much would a waste of 7.5 % be each month? each year? in 10 years? in 20 years?

31. The grocery and meat bill averages \$48 per month for a home. If $12\frac{1}{2}$ % is wasted, what is this for a month? for a year? for 15 years?

32. Richard puts two teaspoonfuls of sugar upon his fruit. If half a teaspoonful is wasted each time what per cent of the sugar does he waste?

33. In a football game the North Junior High School team carried the ball forward 268 yd. and the East team 312 yd. The forward distance made by the North team was what per cent of that made by the East team?

75. Discount.—In order to increase their sales merchants often reduce the prices on their goods a certain per cent. Such reductions are called discounts. The per cent that the price is reduced is called the **rate of discount**. The amount that the price is reduced is the **discount**.

EXERCISES

1. A suit of clothes marked at \$32 is sold at 25 % discount. For how much is the suit sold?
2. A grocer sells corn at 20¢ per can, but gives a discount of 10 % to those buying a dozen or more cans at one time. How much is saved by buying a dozen cans at one time?
3. In February a clothing store reduces the price on its overcoats 15 %. How much do they reduce the price on an overcoat marked \$25? marked \$30? marked \$45?
4. A furniture store closing out a certain kind of dining-room furniture sells it at a discount of 25 %. What will be the price of six chairs that originally sold at \$4.50 each?
5. A certain store gives a discount of 5 % for cash. How much is saved in paying cash for a purchase of \$76.50?
6. After a fire a store sold its stock at a discount of 45 %. What was the discount on a bill amounting to \$37.50?
7. Mr. Williams owns a grocery store. He receives a discount of 3 % on all bills he pays cash. How much must he pay cash for a bill which is \$639.45? How much did he save by paying cash?
8. A piano is marked \$375. It is sold at a discount of 35 % for cash. How much was paid for the piano?
9. State as many reasons as possible why merchants give discounts. Make up reasonable problems for the class to solve that could arise from these causes.
10. What actual cases of discount do you know about, or what discount advertisements have you seen?

11. A wholesale book concern allows its retail dealers a discount of 2 % for cash. Find the amount of the following bill and how much the retail merchant must pay for it in cash:

20 copies of U. S. Histories.....	at \$.95
35 " " Arithmetics.....	at .85
15 " " Readers I.....	at .43
25 " " Readers II.....	at .55
15 " " Drawing Bk. I.....	at .37
Total.....	
Less 2 % for cash.....	_____
Net.....	

12. Make up a bill similar to the above, compute the cost of each item after deducting a discount of 5 %.

76. **Taxes.**—Who pays your teacher? the governor of your State? the President of the United States? Each property owner and those having yearly incomes above certain amounts pay a certain per cent of these amounts as taxes to pay for public improvements and for public services.

EXERCISES

- Find the tax on \$ 2,500 at 2 %; at $1\frac{1}{4}$ %.
- Mr. Jerold has a yearly income of \$ 3,450. He pays an income tax of 1 % of this above \$ 2,160. What is his income tax?
- Mr. Wilmore pays a tax of $1\frac{1}{4}$ % on each of his properties, valued at \$ 2,850, \$ 3,700, and \$ 4,850. What is his total tax?
- The school tax in a certain city is .8 %. How much does a man pay towards the support of the public schools who owns property valued at \$ 5,600? valued at \$ 25,000.

77. **Interest.**—Money is borrowed by paying for its use in the same way that rent is paid for a house. The price paid is usually a certain per cent of the money borrowed. Mr. Ellis may loan \$200 to Mr. Rowland for which he receives 5% of the money loaned. At the end of the year Mr. Rowland pays Mr. Ellis the \$200, which is called the principal. In addition Mr. Rowland pays \$10, or 5% of the \$200. This \$10 is called the interest. The per cent of the principal which is paid for its use, 5% here, is called the rate per cent, or merely the rate of interest. The sum of interest and principal is called amount.

EXERCISES

1. Mr. James borrowed \$500 for one year at 7% interest. What was the principal? the rate of interest? How much interest did he pay at the end of the year?
2. Mr. Myer borrowed \$800 at 7% interest. How much interest did Mr. Myer pay at the end of the year?
3. Harry earned \$35 during vacation and loaned it one year at 6% interest. How much interest did he receive at the end of the year? What was the rate of interest?
4. Working Saturdays and after school, George earned \$85 during the year. He placed it in the savings bank, which paid him 4% interest. How much did he receive in interest at the end of the year? How much money did he then have?
5. Mr. Cross loaned \$850 for one year at 6% interest. Find the interest. Find the amount.



6. How much will \$600 amount to at the end of a year if loaned at 7 % interest? If loaned at 5 % interest?
7. Mr. Williams borrowed \$650 from his bank and paid 8 % interest for one year. How much interest did he pay?
8. If Mr. Williams had borrowed the \$650 at 8 % for one-half year, how much interest would he have paid?
9. Mr. Maxwell borrowed \$800 from his bank at 6 % for six months. How much interest did he pay?
10. Mr. Jason borrowed \$450 from his bank at 8 % for three months. How much interest did he pay?

78. Commission.—Very often a person, as a real-estate agent, carrying out a sale for another is paid a certain per cent of the sale in the place of a salary. Such payments are called **commission**.

EXERCISES

1. A travelling salesman receives 2 % commission on his sales. How much commission will he receive for the month when his sales are \$9,740?
2. A real-estate agent sells a house for \$4,750. How much commission does he receive at the rate of $2\frac{1}{2}\%$? How much does the owner of the property receive?
3. A clerk received a weekly salary of \$8 and $\frac{1}{2}\%$ commission on his sales. How much did he receive the week his sales were \$112? \$145? \$164?
4. How much commission will a real-estate agent receive for selling a property at \$3,550, if the rate of commission is 3 %?
5. Look in the papers for prices of property offered for sale and compute the commission at the rates of your community. Also find the net amount received by the owner.

79. Finding Per Cents.—A per cent of a number is often called the **rate per cent**, or merely the **rate**. The part of a number that is asked for is often called the **percentage**. This is what has been found up to the present.

We shall now find what per cent one number is of another, as 54 is what per cent of 600?

This means, what per cent of 600 is 54; or, as a fraction, what fraction of 600 equals 54? Using the idea of opposites or reverse operations we have:

$$\text{? of } 600 = 54; \text{ or } .? \times 600 = 54.$$

Hence,

$$. ? = \frac{54}{600} = .09 = 9\%.$$

Reread page 25. Pay special attention to the idea of the opposite or reverse process.

EXERCISES

1. What per cent of 400 equals 20?
2. What per cent of 60 equals 30? equals 15?
3. What per cent of 56 is 7? What per cent of 72 is 12?
4. 9 is what per cent of 45? 12 is what per cent of 60?
5. 75 is what per cent of 300? 35 is what per cent of 1,400?
6. In an assignment of 20 problems John worked 18. What per cent of the whole lesson was this?
7. Henry's football team has won 5 and lost 3 games. What per cent of the total number played was won? This is called the team's **percentage**.
8. Find the percentage of the football teams which have won 6 games and lost 5 games; won 4 and lost 6; won 3 and lost 4; won 3 and lost 2; won 2 and lost 3.

Reds	0	0	6	6
Blues	7	3	0	0

10. How many scores did each team make? Reds' score is what per cent of Blues' score? Blues' score is what per cent of Reds' score?
11. What was lost on goods bought for \$3,200 and sold for \$2,800? What per cent of the buying price was lost?
12. Goods bought at \$450 are sold for \$560. How much is gained? What per cent of the buying price is this?
13. A desk marked at \$6.50 is sold at a discount of \$1.50. For what is the desk sold? The discount, \$1.50, is what per cent of the marked price?
14. In a certain school there are 317 girls and 265 boys. What per cent of the total enrollment is girls? is boys? What does the per cent of girls plus the per cent of boys equal? Why is this so?
15. A salesman received $2\frac{1}{2}\%$ commission of his total sales. One week his commission was \$24.50. What had his sales been that week? \$24.50 is $2\frac{1}{2}\%$ of how many dollars?
16. Mr. Robins bought an automobile for \$985 and sold it three years later for \$500. What was the loss? The loss is what per cent of the buying price?
17. In a civil-service examination 35 out of 167 applicants passed the examination. What per cent of all who tried the examination was this?
18. At the above rate how many may be expected to pass out of 125 who try the examination?
19. A dairyman got 9 qt. of cream from 12 gal. milk. What per cent of the milk was cream?

V

MEASURES

80. Abstract Numbers.—Numbers which do not refer to any particular kind of thing are called **abstract**. Such numbers are 5, 79, 80, $\frac{3}{4}$, etc.

81. Concrete Numbers.—Numbers which refer to a particular kind of thing are called **concrete**. Such numbers are 7 men, 3.5 yd., 5 stories, 3 baseballs, and so on.

Which of the following numbers are abstract and which are concrete: 3 boys ? 56.2 ? 3.4 lb. ? 7 dogs ? 793 ? 45 ? 67 ? 29 chickens ? 32 kites ? 8 marbles ?

82. Measures or Denominate Numbers.—When we count we find out **how many** objects of a kind there are, as, 7 apples or 3 pencils. In buying or selling we are not concerned nearly so much about how many apples or pencils as we are about their quality and **how much** there is of apples or of pencils. Can there be more paper in 1 piece of paper than in 3 pieces of paper ? Answers to all such questions of **how much** require measurements. The most common measures are those of length, area, angles, volume, weight, time, and value in the form of money. **Measures** are often called **denominate numbers** because they denote the **kind** of concrete number just as the denominator of a common fraction denotes the **kind** of a fraction. **Denominate numbers** with two or more units, as 2 ft. 7 in., are also called **compound denominate numbers**, or merely **compound numbers**.

83. Tables.—The tables of measures and abbreviations will be found upon pages 225 to 228. Note the carpenters' and architects' abbreviations for foot and inch. Angles, areas, and volumes will not be taken up before chapters VII and VIII, in which geometrical measures will be studied.

84. Quantity.—Denominate numbers and measures of various kinds will often be spoken of hereafter as **quantities**. We speak of the quantity of medicine in a bottle and of the quantity of coal in a bin.

EXERCISES

1. Which of the following are denominate numbers: 3.4 ? 7 lb. ? 5 yd. ? 6 marbles ? 473 ? \$45 ? 67 books ? $\frac{5}{8}$ T. ?
2. Can there be more bread in 2 than in 3 loaves ? more paper in 5 than in 7 sheets ? more corn in 12 than in 17 ears ? more coal in 6 than in 8 loads ?

85. History of Measures.—Our common system of measures, also called the English system, had a very crude beginning. Most of the units, as is seen from their names, have to do with grains or parts of the body. Among the units are foot; hand; finger breadth; span, which is from the tip of the thumb to the tip of the little finger when the hand is outstretched; yard, or *ell*, which is the old English name for arm; the pace, the average length of a step; the mile, from the Latin *millia passuum*, a thousand paces. In old Rome the inch meant $\frac{1}{12}$ of a foot; in France and the Scandinavian countries it meant the length of the first thumb joint; in the British Isles it meant the length of three barley corns placed end to end. The rod was probably the length of some measuring-pole. It is not known just what is the origin of our unit of land measure, the acre, but it is thought to have meant the amount of land a man could plow in a day with a team of oxen.

An amount which would make up into a convenient package may be the origin of our peck. We have no definite history of the gallon, but the quart was likely one-quarter of the gallon.

The pound, the unit of weight, has been of many sizes, but has always been made up from a certain number of grains of some kind, usually barley. Hundredweight is, of course, 100 pounds while 20 hundredweight are called a ton. How many pounds does this make a ton? A stone weighing 14 pounds was used in the British Isles in weighing. Eight of these, or 112 pounds, they called a hundredweight. From this comes the long ton of 2,240 pounds, used only in mines and the custom-house.

Over 4,000 years ago the people of Babylon divided the circle into 360 equal parts, which we call degrees. They made this division because they thought that the sun made a journey around the earth every 360 days. After all these years we are to-day using this convenient division of a circle into degrees.

Month likely refers to the changing of the moon.

86. Reductions Descending.—It is often necessary to change measures from one denomination to another. Reduce 7 yd. 2 ft. 5 in. to inches.

$$\begin{array}{r} 7 \\ \underline{3} \\ 21 \end{array}$$

$$\begin{array}{r} 2 \\ \underline{23} \\ 21 \end{array}$$

$$\begin{array}{r} 5 \\ \underline{23} \\ 21 \end{array}$$

$$23 \text{ ft.} = 7 \text{ yd.} + 2 \text{ ft.} = 7 \times 3 \text{ ft.} + 2 \text{ ft.}$$

$$\begin{array}{r} 12 \\ \underline{46} \\ 23 \end{array}$$

$$\begin{array}{r} 5 \\ \underline{281} \\ 23 \end{array}$$

$$281 \text{ in.} = 23 \text{ ft.} + 5 \text{ in.} = 23 \times 12 \text{ in.} + 5 \text{ in.}$$

87. Reduction Ascending.—Reduce 176 in. to yards, feet, and inches.

$$\begin{array}{r} 12)176 \text{ in.} \\ \underline{3)}14 \text{ ft., } 8 \text{ in.} \\ 4 \text{ yd. } 2 \text{ ft. } 8 \text{ in.} \end{array}$$

Reduce 5 bu. 3 pk. 6 qt. to bushels.

$$1 \text{ bu.} = 32 \text{ qt. } 3 \text{ pk. } 6 \text{ qt.} = 30 \text{ qt. } 30 \text{ qt.} = 1\frac{1}{2} \text{ bu.} = 1\frac{1}{8} \text{ bu.}$$

$$\text{Hence, } 5 \text{ bu. } 3 \text{ pk. } 6 \text{ qt.} = 5\frac{1}{8} \text{ bu.}$$

EXERCISES

Make the following reductions:

- | | |
|--------------------------------|-------------------------------|
| 1. 5 yd. 2 ft. 3 in. to inches | 4. $\frac{3}{4}$ T. to pounds |
| 2. 4 bu. 3 pk. 5 qt. to quarts | 5. 3 gal. to pints |
| 3. 5 gal. 3 qt. to pints | 6. $\frac{5}{6}$ yd. to feet. |

Change the following so as to contain the highest possible denomination:

- | | | |
|--------------|-----------------|--------------------|
| 7. 164 in. | 10. 5,634 oz. | 13. 435 liquid qt. |
| 8. 6,340 lb. | 11. 452 ft. | 14. 6,780 lb. |
| 9. 642 pt. | 12. 546 dry qt. | 15. 245 ft. |

16. How many yards are there in 3 yd. 2 ft. 6 in.? 18 in.? 2 ft. 8 in.? 28 in.? 42 in.? 64 in.? 7 ft.? 11 ft.?

17. How many gallons are there in 6 qt.? 8 pt.? 3 qt.? 1 pt.?

18. How much is made by buying $4\frac{1}{2}$ bu. potatoes at \$1.40 per bushel and selling them at \$.55 per peck?

What is the gain per cent?

19. A case of berries holding 24 qt. is what per cent of a bushel?

20. A train going 40 mi. per hour goes how far in 1 min.?

21. A train going 30 mi. per hour goes how many feet per second? how many yards per minute?

88. Addition and Subtraction of Denominate Numbers.

—To add or subtract denominate numbers write the quantities under each other so that those of the same denomination fall in columns, the lowest to the right.

$$\begin{array}{r}
 & & 1 \\
 & 5 & \text{yd.} & 2 & \text{ft.} & 7 & \text{in.} \\
 1 & & 1 & & 3 & & \\
 4 & & 0 & & 9 & & \\
 3 & & 2 & & 0 & & \\
 \hline
 15 & & 0 & & 7 & &
 \end{array}$$

First add the column to the right, giving 19 in., or 1 ft. 7 in.; write 7 under inches' column. The 1 is carried to the next column just as in adding columns of units and tens. The second column is now added in the same manner.

In subtraction the number is found which added to the subtrahend gives the minuend.

$$\begin{array}{r}
 & & 5 \\
 9 & \text{yd.} & 1 & \text{ft.} & 8 & \text{in.} \\
 3 & & 2 & & 5 & \\
 \hline
 5 & & 2 & & 3 &
 \end{array}
 \quad \text{To 5 in. add 3 in. to make 8 in.; write 3 under inches' column. To 2 ft. add enough to make 1 ft. and 1 yd., that is 2 ft.; write 2 under feet's column. 1 yd. carried and 3 yd. give 4 yd., and 4 yd. added to 5 yds. make 9 yds.; write 5 under yards' column.}$$

EXERCISES

- Find the sum of 3 yd. 2 ft. 7 in.; 4 yd. 1 ft. 9 in.; 2 ft. 11 in.; 2 yd. 6 in.
- Find the sum of 3 bu. 2 pk. 5 qt.; 4 bu. 1 pk. 7 qt.; 2 bu. 6 qt.; 3 pk. 5 qt.
- Find the cost at \$ 5.50 per ton of the following loads of coal: 3,560 lb.; 3,480 lb.; 2,880 lb.; 4,230 lb.
- From 18 gal. 3 qt. take 9 gal. 1 qt. 1 pt.
- Subtract 5 rd. 3 yd. 2 ft. from 11 rd. 2 yd. 1 ft.
- A grocer bought 50 bu. potatoes at \$ 1.40 per bushel. After he had sold 42 bu. $6\frac{1}{2}$ pk. at 60¢ per peck, he sold the remainder at cost. What was his gain? What was his gain per cent?



7. Oilcloth 38 in. wide costs 25 ¢ per yard and that 54 in. wide costs 35 ¢ per yard. What will it cost to cover a kitchen table 2 ft. 4 in. by 3 ft. 2 in. most economically, 3 in. being used to turn over each edge?
8. What is your exact age in years, months, and days?
9. What will be your exact age next Christmas? next Memorial Day? next Thanksgiving Day?

89. Multiplication of Denominate Numbers.—Denominate numbers can be multiplied by any abstract number.

$$\begin{array}{r}
 5 \text{ gal. } 3 \text{ qt. } 1 \text{ pt.} \\
 \times \quad \quad \quad \quad \quad 9 \\
 \hline
 52 \quad 3 \quad 1
 \end{array}
 \qquad
 \begin{aligned}
 9 \times 1 \text{ pt.} &= 9 \text{ pt., or } 4 \text{ qt. } 1 \text{ pt.; } 1 \text{ is placed} \\
 &\text{under pints' column. } 4 \text{ qt. added to } 9 \times 3 \text{ qt.} \\
 &\text{make } 31 \text{ qt., or } 7 \text{ gal. } 3 \text{ qt.; write } 3 \text{ under} \\
 &\text{quarts' column. } 7 \text{ gal. added to } 9 \times 5 \text{ gal. make} \\
 &52 \text{ gal.}
 \end{aligned}$$

EXERCISES

- Multiply 3 rd. 4 yd. 2 ft. 7 in. by 4; by 8; by 5.
- Multiply 2 T. 3 cwt. 53 lb. by 5; by 14; by 7.
- Moulding $\frac{7}{8}$ in. wide is used to frame six pictures each 8 in. by 3 in. The matting between the picture and the frame is $1\frac{1}{4}$ in. How many feet of moulding are needed?
Draw a diagram.
- What will 8 sash curtains each 5 ft. 9 in. long cost at 28 ¢ per yard?
- Find the cost of a border at 14 ¢ per yard used in papering a room 14 ft. 7 in. by 16 ft. 8 in.
- A gardener last year raised from a certain plot of ground 5 bu. 6 qt. of peas. This year by giving it more attention he expects to increase the yield 40 %. How much would the crop this year then be? What would be the gain in money if peas sell at 8 ¢ per quart?

90. Division of Denominate Numbers.—Two cases of division of denominate numbers arise. How much will there be in each of three groups from 5 bu. 3 pk. 6 qt.?

$$3) \underline{5 \text{ bu. } 3 \text{ pk. } 6 \text{ qt.}} \\ \quad \quad \quad 1 \quad 3 \quad 7\frac{1}{3}$$

Begin with the largest denomination; 5 bu. \div 3 give a quotient of 1 and a remainder of 2 bu. The 2 bu. and the 3 pk. make 11 pk., which divided by 3 give 3 pk. for quotient and a remainder of 2 pk. The 2 pk. and .6 qt. make 22 qt., which divided by 3 give $7\frac{1}{3}$ qt.

The second case is that of finding how many groups and fractional parts of a group of a given size can be made out of a given denominate number. For instance, how many groups of 1 bu. 2 pk. 2 qt. can be made from 3 bu. 2 pk. 5 qt.? Each is reduced to quarts and the latter divided by the former. Thus:

$$\frac{117}{50} = 2\frac{17}{50}$$

EXERCISES

1. Divide 8 yd. 2 ft. 5 in. by 3; by 4; by 12.
2. Divide 42 bu. 3 pk. 6 qt. by 5; by 8; by 15.
3. Divide 8 gal. 2 qt. into equal amounts of 2 qt. 1 pt.
4. How many lengths of 2 ft. 4 in. are there in 3 rd. 4 yd. 2 ft.?
5. John, Myron, and Harry gathered 5 pk. of walnuts. If they divide the nuts equally, how much will each boy get?
6. In a certain city there are 5 mi. of street railway. How many rails each 30 ft. have been used in the construction?
7. A man travelling in an automobile six days a week averages 8 gal. 1 qt. of gasoline per day the first week. The second week he averages 7 gal. 1 qt. per day, and the third week he averages 6 gal. 3 qt. per day. What is the total cost at 21.7 cents per gallon? What is the cost per day?

8. How many pounds of sugar sold at 11 lb. for \$1.00 will a farmer receive at the grocery in exchange for 15 doz. eggs at 35¢ per dozen ?
9. Mary was instructed to purchase enough ribbon for a class of fifty pupils in a school whose colors are olive-green, and orange. If each ribbon is to be $8\frac{1}{2}$ in. long, how many yards of each must Mary purchase ?
10. Five boys were hired by a gardener to gather the beans from a "patch" consisting of 63 rows each $17\frac{1}{2}$ rd. long. What length of row should each boy pick ?
11. A rug 9 ft. by 12 ft. is to be placed upon a floor 14 ft. by 16 ft. It is desirable to paint the exposed part of the floor. If a margin of 3 in. is to be allowed under the rug, how wide a strip must be painted on the sides and on the ends ?
12. A family uses $1\frac{1}{2}$ qt. of milk per day. At the rate of $9\frac{1}{2}$ qt. for \$1, what will be the milk bill during October ? during January ? during February ?
13. Which is the faster train, one going 30 mi. per hour or one going 48 ft. per second ? How much farther will the faster train go in 1 hr. ? in 5 hr. ? in $4\frac{1}{2}$ hr. ?
14. In the fall a grain dealer bought 12,850 lb. of ear corn, 75 lb. to the bushel, at 95¢. How many bushels did he buy and what was the total cost ? He estimated that the corn would shrink 15 % of its weight by spring. How many bushels would he then have at 70 lb. to the bushel ? At how much must he sell it to gain 20 % on the money invested ? What is that per bushel ?
15. Idaho Springs, Colorado, is 7,536 ft. above sea level. Express this in miles and a fraction.
16. How many fluid ounces are there in 1 pt. ? 3 pt. ?
17. Play a number game, using denominative numbers.

91. Metric System.—At the end of the eighteenth century a group of the best scientific men in France was appointed to make up a better system of measures. They invented the simple metric system. Each unit is ten times that of the next lower one, thus giving only one very simple multiplier to remember in the place of the many difficult ones in the English system. On page 81 it will be learned that reductions can be made by merely moving the decimal point, because the metric system has only this one multiplier, ten. Only five tables need to be learned and three of these are wholly alike, except for one word.

The tables are not difficult to learn, especially if it is noticed how similar the first half of each is to the table of United States money. This is shown by a glance at the tables on pages 228 and 229. Learn and use these tables as necessary. Adoption of the metric system will not result in any great confusion, as many believe, because the metric units are so nearly equal to those of the English system. This is clearly shown on page 83. Expenses in making the change will not be nearly as great as is feared. Scales need only to have their beams and weights made over. Factories may be permitted to use the same patterns, moulds, and so on, until they are worn out or replaced by those of other patterns. Adoption of the metric system will mean a saving of time and money, and not a waste, as so many fear.

Scientific men the world over have adopted the metric system because of its simplicity. It has also been adopted exclusively in most countries except China, Japan, England, and the United States. It is now used almost altogether in our government measurements; even the yard is defined in terms of the meter. In 1866 the United States made it legal to use the metric system of measures and it should now be made the only system.

92. Linear Measure.

—This little girl who is going to France wishes to find how much more ribbon she will get in buying a meter than in buying a yard of ribbon. She measures a meter stick with a tape measure and finds that the meter is about 3 in. longer than a yard.

**EXERCISES**

1. 3 in. is what part of a yard ? If a meter is 3 in. longer than a yard, what part of a yard is a meter longer than a yard ?
2. 3 in. is what part of 39 in. ? If a meter is 39 in. long, what part of this is the meter longer than the yard ?
3. What should the girl pay for ribbons by the meter that sell for 72 ¢ per yard ? for \$ 1.08 per yard ?
4. What should she pay for ribbons by the yard that sell for 42 ¢ per meter ? for 65 ¢ per meter ?
5. Give the United States money table. Give the metric table for length. Be sure to notice the similarity.
6. Margaret's uncle has sent her from Naples, Italy, a string of coral beads $\frac{2}{3}$ of a meter long. About how many inches is this ? This is almost what part of a yard ?
7. Find your height in inches. Express this to 2 decimal places in meters, calling a meter 39 in.

93. Weight.—Harriet and Jason are visiting their uncle in Cuba. The lady is showing them two candy-boxes. One box holds a pound and the other holds half a kilogram. The kilogram equals about 2.2 lb.



EXERCISES

1. If the larger box that the lady has holds $\frac{1}{2}$ Kg., how many pounds does it hold?
2. Which box is the more economical for these children to buy, a pound box at 60¢ or a $\frac{1}{2}$ Kg. box at 65¢?
3. About how many kilograms would you ask for in buying 10 lb.? 6 lb.? 25 lb.? 100 lb.? 36 lb.?
4. The metric ton contains about 2,200 lb. How much is this over one ton?
5. The metric ton is what part of our ton more than our ton? Is the metric ton equal to 1.1 of our ton?
6. When coal sells for \$6 per ton, what should a metric ton cost?
7. How many metric tons of coal would equal about 6 tons? 10 tons? 25 tons?



94. Capacity.—Harriet and Jason in going on a picnic filled a quart thermos bottle from a liter bottle of chocolate. The picture shows that the liter bottle held a little more than a quart. A liter equals 1.0567 qt. What would a liter of milk cost that sells for 12¢ per quart? for 15¢ per quart? for 8¢ per quart?

95. Reductions.—Reductions in the metric system are very much simpler than reduction in the English system.

Reduce 5 Km. 3 Hm. 8 Dm. 4 dm. 7 cm. to m.; to cm.

We need only note that 3 Hm. is .3 Km., that 8 Dm. is .08 Km., that 4 dm. is .0004 Km., and that 7 cm. is .00007 Km. in order to write at once 5.38047 Km. for 5 Km. 3 Hm. 8 Dm. 4 dm. 7 cm. As there are no meters its place is filled by a zero. By properly placing the decimal point in this, 5.38047 Km. = 538.047 Dm. = 5,380.47 m. = 538,047 cm.

Explain these last reductions fully.

All reductions should be made as the above by writing down the units one after the other, without carrying out the multiplications by 10, placing zeros for any omitted, and then properly placing the decimal point.

EXERCISES

Write out the following without making any computations:

1. Change 65.38 Dl. to liters; to hektoliters.
2. In 6 Km. 7 Hm. 8 m. 5 dm. 9 cm. there are how many meters? centimeters?
3. In 7 Kg. 8 Hg. 2 g. 8 dg. 4 cg. there are how many grams? kilograms?
4. Reduce 7 Kg. 2 Dg. 9 g. 3 cg. to hektograms; to grams; to decigrams.
5. Reduce 6 Km. 4 Hm .8 m. 9 dm. to kilometers; to meters; to centimeters.
6. Reduce 6,478.45 Kg. to metric tons; to grams.
7. Compare the amount of work in the above exercises with that of the first 14 exercises on page 73.

96. Four Fundamental Operations.—In carrying out the four operations of addition, subtraction, multiplication, and division with metric units it is best first to reduce each to the **same unit**. The operations can then be carried out just as with decimals or other concrete numbers, as United States money.

From 9 Km. 5 Hm. 2 dm. take 3 Km. 6 Dm. 9 m. 3 cm.

9500.2 m. Minuend reduced to m.

3069.03 Subtrahend reduced to m.

6431.17 m. Subtracting as in decimals.

How many lengths of .75 m. can be cut from a length of 2 Dm. 3 m. 4 dm.?

$$\begin{array}{r} 31.2 \\ \hline 75)2340 \\ \underline{225} \\ 90 \\ \underline{75} \\ 15 \\ \underline{15} \end{array}$$

2 Dm. 3 m. 4 dm. reduces to 23.4 m., the same denomination as the divisor. By what are divisor and dividend multiplied to make each a whole number? Explain the abbreviated division.

EXERCISES

1. Add 2,434.62 m., 546.07 m., 105.73 m., and 103.047 m.
2. Add 8 Km. 5 Hm. 9 Dm. 3 m. 2 dm.; 2 Km. 4 Dm. 7 m.; 2 Hm. 8 Dm. 6 m.; 7 Km. 9 Dm. 8 m. 7 cm. 9 mm.; 8 Km. 7 Hm. 9 m. 6 dm. 8 mm.
3. From 9 Km. 6 Hm. 3 Dm. 2 m. 5 cm. take 3 Km. 8 Hm. 5 Dm. 7 m. 2 dm.
4. Multiply 456.74 g. by 45.32.
5. Divide 3,205.74 m. by 54.6.
6. How many times can 24.5 g. be taken from 7 Hg. 5 Dg. 4 g.?

7. How many metric tons are there in each of the following: 678 Kg.; 437 Kg.; 308 Kg.; 8,503 Dg.; 30,456 Kg.
8. What is the cost of 5 Dm. 7 m. 8 dm. cloth at \$2.35 per meter?
9. What is the value of 6.473 Hl. milk at 12 ¢ per liter?
10. How long will it take to ride 9,834.7 Dm. at the rate of 31 Km. per hour?

97. Equivalents of Metric Units in English Units.—

$$\begin{aligned}1 \text{ m.} &= 1.1 \text{ yd.} \\1 \text{ Kg.} &= 2.2 \text{ lb.} \\1 \text{ metric ton} &= 1.1 \text{ T.} \\1 \text{ l.} &= 1 \text{ qt.}\end{aligned}$$

EXERCISES

State without the use of pencil and paper about what amount must be bought in metric weights or measures to get each of the following:

1. 6 lb.; 12 lb.; 25 lb.; 100 lb.; 36 lb.; 16 lb.
2. 5 T.; 8 T.; 32 T.; 56 T.; 100 T.
3. 6 yd.; 12 yd.; 5 yd.; 40 yd.; 8 yd.
4. 2 qt.; 6 qt.; 3 gal.; 8 gal.; 2 gal.; 3 qt.
5. The sum of the metric tons in Ex. 7, above, is equal to about how many of our tons?
6. Mary's father gave her 5 yd. of cloth for a dress. About how many meters was this?
7. Coal sold last year for \$4.50 per ton. About what would this be per ton after the metric system has been adopted?
8. What will be the cost per meter of cloth selling for 35 ¢ per yard? for 60 ¢ per yard? for \$1.25 per yard?
9. Mr. Brown bought a 40 ft. lot at \$20 per foot. At what must he sell it per meter to gain 20 %?

VI

LITERAL NUMBERS

98. Arithmetical Equation.—Mathematicians use a sort of shorthand in making statements. These statements consist of mathematical symbols and letters. The letters taking the places of words are usually the first of the word. Thus, in the place of

Minuend minus subtrahend equals remainder
is written M — S = R (1)

Similarly,

total price equals number of articles times price of one.

P = n \times p (2)

Each expression, as (1) and (2), is called an **equation**. It states that two numbers, or sets of numbers, **are equal**; that is, they have the same value. The parts on each side of the equals sign are called the **members** of the equation. Every letter in the equation represents some number.

An equation shows at a glance the relation between the numbers it contains. Equations will be used to simplify the solution of problems. Hence, learn thoroughly how to make and to use these shorthand statements.

To express that Henry is 15 yr. older than George, write

$$A_H = A_G + 15.$$

The small H and G tell whose age is meant. Small letters or numerals placed at the lower right-hand corner are called **subscripts**.

EXERCISES

Write each of the following statements in full. Then rewrite them in this shorthand mathematics. State the right and the left hand members of each equation.

1. Minuend equals remainder plus subtrahend.
2. Dividend equals divisor times quotient plus remainder.
(Use D and d for dividend and divisor.)
3. Dividend minus divisor times quotient equals remainder.
4. The larger of two numbers equals the smaller plus six.
(Use N and n .)
5. The larger of two numbers is four times the smaller.
6. The larger of two numbers less the smaller equals five times the smaller number.

Express each of the following as an equation:

7. The larger of two numbers exceeds twice the smaller by 13.
8. Profits equal total sales less total expenses.
9. Total sales less profits equal total expenses.
10. Total sales less profits less expenses equal nothing.
11. Total profits equal total sales times rate of gain.
12. Loss equals total expenses less total sales.
13. Give the equations for losses corresponding to the statements 9, 10, and 11 for profits.
14. Mary is twice as old as Jane.
15. Henry is 5 yr. older than William.
16. Elizabeth is 6 yr. more than twice as old as May.
17. James weighs twice as much as Harry.
18. John weighs 6 lb. less than twice as much as Harry.
19. William weighs as much as Henry and John together.
20. The numerator of a fraction is 3 more than 5 times the denominator.

99. Evaluations.—Replacing letters by their numerical values in an equation is called **evaluation** or **substitution**. Thus, if in

$$D = d \times q + r \quad (1)$$

the quotient is 4, divisor 7, and remainder 3, the dividend is found by replacing the letters in the equation by these numerical values when

$$D = 7 \times 4 + 3 \quad (2)$$

$$= 28 + 3 = 31. \quad (3)$$

EXERCISES

1. Find the dividend when the quotient is 8, divisor 6, and remainder 4; quotient 17, divisor 9, and remainder 7.

2. In division we also have

$$r = D - d \times q.$$

Find r when $D = 532$, $d = 25$, and $q = 21$.

If N and n are two numbers, what is the equivalent English expression of the equations:

3. $N = n + 12.$

5. $N = 5 \times n + 7.$

4. $N = 3 \times n.$

6. $N = \frac{1}{3} \times n + 56.$

7. Find N for each equation when $n = 6$; $n = 15$.

8. What would the letters M , S , and R stand for in subtraction? Fill in the right-hand members of the following equations using these meanings of the letters:

$$M = \dots; S = \dots; R = \dots$$

9. Find M when $S = 17$ and $R = 326$; when $S = 307$ and $R = 137$; when $S = 289$ and $R = 176$.

10. Find S when $M = 635$ and $R = 269$; when $M = 135$ and $R = 79$; when $M = 402$ and $R = 137$.

11. Find R when $M = 735$ and $S = 268$.

12. Play a number game, using equations and substitutions.

100. Solution of Equations.—If an equation contains only one literal number, its numerical value can be found. Finding this value is called **solving the equation**. The numerical value of the literal number found is called the root of the equation.

The equation stating that five times a number equals 30 is

$$5 \times n = 30. \quad (1)$$

Hereafter $5 \times n$, $7 \times n$, etc., will be written $5n$, $7n$, etc. $5n$, $7n$, etc., mean that there are 5 n 's, 7 n 's, etc.

The above equation is then

$$5n = 30. \quad (2)$$

Solutions of equations depend upon the following: 1. *Both members of an equation may be multiplied or divided by the same number and still leave a true equation.* 2. *The same number may be added to, or subtracted from, both members of an equation and still leave a true equation.*

The left-hand member is $5n$. If this be divided by 5 the quotient will be n . Why? This will equal the right-hand member also divided by 5. Hence,

$$n = 6. \quad (3)$$

EXERCISES

1. How much is 30 times any number divided by 3 ? divided by 2 ? divided by 5 ? divided by 10 ? divided by 30 ?

2. How much is $30n \div 3$? $30n \div 2$? $30n \div 10$? $30n \div 5$? $30n \div 30$?

3. By what would you divide $12n$ to get $3n$? to get $4n$? to get $6n$? to get $2n$? to get n ?

Solve the following equations:

- | | | |
|----------------|--------------|----------------|
| 4. $5n = 35$ | 7. $7k = 42$ | 10. $17m = 34$ |
| 5. $7g = 35$ | 8. $3R = 45$ | 11. $5S = 17$ |
| 6. $11b = 132$ | 9. $6T = 48$ | 12. $7A = 36$ |

101. Solution of Equations.—The equation says that 4 times a number, n , added to 3 is as large as 2 times the number added to 7. The balance scale also says the same thing.

$$4n + 3 = 2n + 7$$



If the same number of blocks be taken from each pan it will also balance. All the blocks are taken from one pan and the same number from the other pan. This leaves the pans as in the picture to the right. The number of blocks and weights in each pan is shown by the equation above the scales. If the known weights in the pan with the blocks be removed

$$2n + 3 = 7$$



$$2n = 4$$



shown. The corresponding equation is like the equations solved on the last page. How many known weights does each block weigh?

and a like number from the other pan, the balance is still preserved. One pan now contains only blocks and the other pan only known weights as here

In case a number is subtracted in one member of an equation that number must be added to both members of the equation. Thus,

$$7h - 8 = 3h + 4 \quad (4)$$

$$7h = 3h + 12 \quad (5) \quad 8 \text{ added to both members.}$$

$$\cdot 4h = 12. \quad (6) \quad 3h \text{ subtracted from both members.}$$

Hence, $h = 3.$ (7) How?

102. Checking Solutions.—The solution of an equation is not complete before it has been checked. This is done by substituting the root in the very first equation. If the work is correct, this will produce a true equation; if the work is incorrect, the resulting equation will not be true. Substituting 3 for h in (4) gives,

$$7 \times 3 - 8 = 3 \times 3 + 4 \quad (8)$$

$$21 - 8 = 9 + 4 \quad (9)$$

$$13 = 13. \quad (10)$$

What is the conclusion? If 7 had been found as the root then,

$$7 \times 7 - 8 = 3 \times 7 + 4 \quad (11)$$

$$49 - 8 = 21 + 4 \quad (12)$$

$$41 = 25. \quad (13)$$

What is the conclusion?

EXERCISES

Solve the following equations and check results:

1. $5n = 20$

7. $14Q + 9 = 37$

2. $7k = 35$

8. $11h = 8h + 15$

3. $3m + 4 = 25$

9. $17t = 15t + 18$

4. $\frac{3}{5}L = 21$

10. $5R - 6 = 24$

5. $7g + 3 = 38$

11. $4S - 7 = 5$

6. $\frac{4}{3}x = 32$

12. $7J + 5 = 3J + 29$

- | | | | |
|-----|------------------------|-----|-----------------------|
| 13. | $9T = 2T + 56$ | 23. | $5D + 8 = 3D + 26$ |
| 14. | $.5X = 45$ | 24. | $7S - 6 = 4S + 9$ |
| 15. | $1.2V = 3.6$ | 25. | $9A = 28 - 5A$ |
| 16. | $.6E + 5 = 11$ | 26. | $15D = 68 - 2D$ |
| 17. | $\frac{2}{3}F + 3 = 7$ | 27. | $35M - 15 = 3M + 81$ |
| 18. | $\frac{3}{4}N - 7 = 5$ | 28. | $47B + 34 = 316$ |
| 19. | $.4G + 3 = 19$ | 29. | $34W - 17 = 28W + 31$ |
| 20. | $\frac{1}{3}K - 5 = 6$ | 30. | $.6Y + 5 = .4Y + 7$ |
| 21. | $3T + 11 = 2T + 19$ | 31. | $1.8R + 1 = 1.3Y + 6$ |
| 22. | $6U - 19 = 11$ | 32. | $15H - 14 = 8H$ |

103. Solution of Problems.—First read the statement of the problem very carefully. Next write the same thing in symbols. If necessary, write down the form corresponding to (1) given below, before trying to write the equation. Test all results.

A lawyer bought an office-chair, a desk, and a rug for \$91. For the desk he paid four times as much as he paid for the chair and twice as much for the rug as for the desk. What did he pay for each?

$$\text{price chair} + \text{price desk} + \text{price rug} = \$91 \quad (1)$$

$$c + 4c + 8c = \$91 \quad (2)$$

$$13c = \$91 \quad (3)$$

$$c = \$91 \div 13$$

$$\left. \begin{array}{l} c = \$7 \\ 4c = \$28 \\ 8c = \$56 \end{array} \right\} \quad (4)$$

$$\begin{aligned} \text{Check: } & \$7 + 4 \times \$7 + 8 \times \$7 = \$91 \\ & \$7 + \$28 + \$56 = \$91 \\ & \$91 = \$91 \end{aligned} \quad \left. \right\} \quad (5)$$

What is the conclusion?

Find the principal which will amount to \$ 784 in two years at 6 %.

$$\text{principal} + \text{interest} \quad \text{equals} \quad \$ 784 \quad (1)$$

$$p + 2 \times .06 \times p = \$ 784 \quad (2)$$

$$p + .12p = \$ 784 \quad (3)$$

$$1.12p = \$ 784 \quad (4)$$

$$\text{Hence,} \quad p = \$ 784 \div 1.12 \quad (5)$$

$$p = \$ 700 \quad (6)$$

$$\text{Check: } \$ 700 + 2 \times .06 \times \$ 700 = \$ 784 \quad (7)$$

$$\$ 700 + \$ 84 = \$ 784 \quad (8)$$

What is the conclusion?

Study the solutions of the last two problems and the ones on pages 87, 88, and 89 very carefully before trying to solve any of the exercises.

EXERCISES

1. Five times a number less two times the number is 36.
Find the number.
2. 2.5 times a number is 75. What is the number?
3. What number multiplied by 15 will give 225?
4. A number and .4 of the number added together will equal 280. What is the number?
5. The sum of a number and twice the number is 12.
Find the number.
6. The sum of two numbers is 48. One number is three times the other number. Find the two numbers.
7. Four times a number equals twice the number and 16 more. Find the number.
8. One number is four times another number. Find the two numbers if their difference is 15.
9. The difference between two numbers is 35. One number is six times the other. Find the numbers.

10. Mr. Smith bought a lot and built a house upon it. The house cost five times as much as the lot. If both the lot and the house cost him \$2,700, what was the cost of each?

11. James bought a tablet and two books. He paid three times as much for one book as he did for the tablet and eight times as much for the other book as he did for the tablet. If his total bill was \$1.80, what did he pay for each?

12. John, William, and Henry are paid \$2.45 for doing some work. John receives twice as much as William and William receives twice as much as Henry. How much does each receive?

13. Josephine is three times as old as Marjory. How old is each if the sum of their ages is 20 yr.?

14. William is $\frac{2}{3}$ as old as John. How old is each if the sum of their ages is 50 yr.?

15. Mary is seven times as old as Jane. How old is each if the sum of their ages is 48 yr.?

16. Cora, who was asked how old she was, replied: "If you add seven years to three times my age that will equal my age plus twenty-nine years." How old was Cora?

17. In a schoolroom of 36 pupils there are $\frac{1}{4}$ as many boys as there are girls. How many boys and how many girls are there?

18. State as an equation that the numerator of a fraction is five less than the denominator.

19. State as an equation that the denominator of a fraction is seven more than the numerator.

20. Find the fraction from Ex. 19 for which the numerator is 9; 11; 3; 15; 4.

21. William bought a sled after Christmas for \$3.20, which was $\frac{1}{5}$ of the original price. What was the original price?

22. Mary bought a pair of skates after Christmas for \$2.50, which was $\frac{1}{6}$ less than the original price. What was the original price?

23. John bought a pair of shoes for \$4.50, which was $12\frac{1}{2}\%$ above the price of last year. What was the price last year?

24. Set up the general equation saying that one number, N , is a certain part, p , of another number, n .

25. Set up the equation stating that the price now, pn , of an article is a certain fractional part, f , of its price last year, pl .

26. A clerk's salary is \$1,200 for the present year, which is 120% of what it was last year. What was his salary last year?

27. A salesman's salary this year is 27% more than it was last year. What was his salary last year if it is \$2,527.30 this year?

28. What principal at 6% will amount to \$848 in one year?

29. At 7%, what principal will amount to \$570 in two years?

30. At 5.5% interest, what principal will amount to \$2,330 in three years?

31. Solve Exs. 15, 16, and 17, page 26, by using equations.

32. Solve Ex. 10, page 44 and Ex. 19 page 45, by using equations.

33. Solve Exs. 1, 3, 5, page 68, by using equations.

34. Solve Exs. 15 and 18, page 69, by using equations.



104. Coefficients.—As mentioned before, $3n$ is written in the place of $3 \times n$, or $n + n + n$. Such numbers as 3 are called **coefficients**. They tell how many times a number has been added to itself. If no coefficient is given, as in w , one is understood.

Multiplication signs are replaced by a dot midway up the line or omitted altogether. Thus, $a \times v = a \cdot v$, or av .

105. Exponents.—A number multiplied by itself is not written nnn but n^3 ; not dd but d^2 ; not $kkkkk$ but k^5 . The small number placed to the right and above another number tells how many times the latter number is taken as a multiplier. The small number is called the **exponent** of the other. It is also called the **power**, **degree**, or **index**. A^2 is read "A square." A^3 is read "A cube." A^5 is read "A raised to the 5th power," or "A with an exponent 5," or "A with the index 5," or "A raised to the 5th degree," or "A fifth."

106. Numerical and Literal Expressions.—A statement in numerals and symbols is called a **numerical expression**; $5 + 2 \times 3$. A statement in letters and symbols is called a **literal expression**; $3mn + h^3 - 7t^2k^3$.

107. Terms.—A literal expression which contains neither plus nor minus, and each part of a literal expression which is between the signs + or - is called a **term**. In $4ar^2 - 5mn^3 + 6a^3b^2c$ the terms are $4ar^2$, $5mn^3$, and $6a^3b^2c$.

Expressions of only a single term are called **monomials**; $7an^3p^2$. Expressions of two terms are called **binomials**; $3g^2h^3 + 9s^5w^3$. Expressions of three terms are called **trinomials**; $2c^6d^2 - 7t^3r + x^7y^3$. Expressions of several terms are called **polynomials**. Write one of each.

An expression as $9a^5b^3 + 7c^4d^2 - 14e^6f^3$ is read: "Nine a fifth b cube, plus, seven c fourth d square, minus, fourteen e sixth f cube."

EXERCISES

Name the terms, coefficients, and exponents in the following. Read the expressions:

- | | |
|---------------------------|-------------------------------|
| 1. $3a^2 + 5a^3r - 4s$ | 4. $29vr^3 - 7c^4b^2 + 9m^7$ |
| 2. $8x^2 - 3k^7h^3 + 6gh$ | 5. $38a^2b^7 - 15c + er^3$ |
| 3. $25cm^3 - 7b^3c^2$ | 6. $18bc^6 + 8c^3 - 19m^2R^3$ |

Correct any errors in the following:

- | | |
|--|---|
| 7. $m + m + m + m = 4m$ | 12. $T \times S \times S \times S = TS^2$ |
| 8. $w + w + w + w = 5w$ | 13. $r \times r \times r = r^3$ |
| 9. $m \times p \times p = mp^2$ | 14. $h \times h \times h = 3h$ |
| 10. $q \times q \times q \times q = q^2$ | 15. $R \times R \times R = 3R$ |
| 11. $n + n + n + n = n^4$ | 16. $m + m + m = m^3$ |

Write the following in symbols:

17. Seven, M sixth power, k cube.
18. Six, m seventh power, h .
19. Twenty, a eighth power, g fourth power.
20. Four, a cube, b sixth power, plus, k ninth power.
21. Seven, x square, minus, four z third power.
22. Sixteen, h cube, plus, m eleventh power.
23. Nine, R eighth power, q square, minus, t cube.

Give the numerical value of the following:

- | | | | |
|-----------|------------|---------------------|-----------------------------|
| 24. 5^2 | 29. 3^4 | 34. $5^2 \times 4$ | 39. $2 \times 6^2 \times 5$ |
| 25. 4^3 | 30. 10^3 | 35. 10×6^2 | 40. $5 \times 6^2 \times 4$ |
| 26. 7^2 | 31. 2^5 | 36. 11×3^3 | 41. $6 \times 2^3 \times 5$ |
| 27. 3^4 | 32. 6^3 | 37. 8×5^2 | 42. $4 \times 2^3 \times 3$ |
| 28. 6^2 | 33. 5^3 | 38. 6×2^4 | 43. $5 \times 7^2 \times 4$ |

Write equivalent values of the following so as to use the largest exponent possible:

- | | | | |
|--------|--------|---------|-----------|
| 44. 9 | 46. 81 | 48. 125 | 50. 49 |
| 45. 25 | 47. 32 | 49. 243 | 51. 1,000 |
| | | | 52. 64 |
| | | | 53. 500 |

108. Like Terms.—Terms which have the same letters raised to the same powers are called **like terms**. Such are, $5a^3v^2$, $937a^3v^2$, $617a^3v^2$; and $9.83Hk^7$, $3Hk^7$; and so on.

109. Addition and Subtraction of Literal Numbers.—Like terms are added or subtracted just as denominative numbers of the same denomination, or any similar concrete numbers. In adding or subtracting literal numbers the coefficients are added or subtracted and the result placed before the literal part. Thus:

$$7a^3m^2 - 5a^3m^2 + 9a^3m^2 = 11a^3m^2,$$

just as, 7 bu. — 5 bu. + 9 bu. = 11 bu.,

or 7 apples — 5 apples + 9 apples = 11 apples.

EXERCISES

Carry out the following operations:

- | | |
|----------------------------------|----------------------------------|
| 1. $3a^4 + 5a^4$ | 7. $3h^2 + 7h^2 - 9h^2$ |
| 2. $5m^3n - m^3n$ | 8. $13R^2 - 8R^2 + 5R^2$ |
| 3. $6a^3b^3 - 5a^3b^3 + 8a^3b^3$ | 9. $25L - 13L + 32L - 7L$ |
| 4. $9h^2k - 4h^2k - h^2k$ | 10. $7gt^2 + 8gt^2 - 15gt^2$ |
| 5. $7ab^2 + 9ab^2 - 15ab^2$ | 11. $9m^7 + 11m^7 - 8m^7 + 3m^7$ |
| 6. $17m^2r^5 + 6m^2r^5 - m^2r^5$ | 12. $18R^3 + 5R^3 - 23R^3 + R^3$ |
13. Find the value of R^2 in $5R^2 + 3R^2 - 4R^2 = 6 + 7 - 5$.
14. Find the value of mn^3 in $7mn^3 - mn^3 + 4mn^3 = 12 - 7 + 15$.
15. Find the value of gt^2 in $13gt^2 - 8gt^2 - gt^2 = 11 + 9 - 4$.

110. Multiplication of Literal Numbers.—In multiplication of one literal number by another, coefficients, letters, and exponents must be considered.

1. Just as 5×4 qt. = 20 qt., so $5 \times 4cv^2 = 20cv^2$. In multiplication, coefficients are multiplied together for the coefficient of the product.

2. Every letter in multiplicand and multiplier will be present in the product.
3. As $n \times n \times n = n^3$ and $n \times n = n^2$, then $n^3 \times n^2 = n \times n \times n \times n \times n = n^5$. That is, the exponent of any letter in the product equals the sum of its exponents in the multiplier and multiplicand. Thus,

$$5b^3n^2 \times 3b^4n^7 = 15b^7n^9;$$

$$7 \times 4a^2cd^6 \times 2a^3d = 56a^5cd^6.$$

EXERCISES

Give the following products, using the simplest order in finding the product of the coefficients:

- | | |
|------------------------------------|---|
| 1. $12 \times 4m^3n^5$ | 11. $5 \times 7a^2b^3 \times 2a^4b^5$ |
| 2. $7gh^7 \times 8g^2h^5$ | 12. $3 \times 4mn^3 \times 5m^3n^7$ |
| 3. $q^6r^3 \times 5qr^6$ | 13. $6jk^2 \times 5j^3 \times 2k^3$ |
| 4. $6m^2n^3 \times 9mn^2$ | 14. $2ab \times 5a^2c \times 4b^6c^3$ |
| 5. $12qr^6 \times 5q^2r^3$ | 15. $4c^2d \times 3d^3f \times 8cf^3$ |
| 6. $7xy^3 \times 8x^9y^5$ | 16. $7m^3n \times nk^3 \times 9m^2k^4$ |
| 7. $17 \times 2a^3d^7 \times 5s^2$ | 17. $8w^2r^3 \times vr^5 \times 7wv^7r^2$ |
| 8. $32tr^2 \times 5tg^6$ | 18. $37ak^6 \times 5bk^5 \times 2a^3b^2$ |
| 9. $7f^2h \times 5f^3$ | 19. $stp^4 \times qr^2 \times 7t^3r^3$ |
| 10. $4m^4n^3j^2 \times 9nj^5$ | 20. $9gh^3r \times 5h^3r^2 \times 8h^5$ |

21. The distance a body falls, due to the earth's pull upon it, is 16 times the square of the number of seconds it has been falling. Use t for the time and express this in symbols.

22. Express in symbols, the area of a rectangle equals its width times its length.

23. Express in symbols, the area of a circle is 3.1416 times the square of the radius.

24. Express in symbols, the volume of a rectangular solid equals the product of its length, width, and height.

25. Play a number game, using addition, subtraction, and multiplication of literal numbers.

111. Division of Literal Numbers.—Division of literal numbers is the opposite of multiplication just as division has always been the opposite of multiplication. There are three parts to be decided upon in division, just as mentioned for multiplication upon pages 96 and 97; the coefficient, letters, and their exponents. Thus, $15k^7h^3 \div 5k^3h^2$ means finding the term which multiplied by $5k^3h^2$ gives $15k^7h^3$. The coefficient is 3, since $5 \times 3 = 15$; the exponent of k is 4, since $3 + 4 = 7$; the exponent of h is 1, since $2 + 1 = 3$. Hence, $15k^7h^3 \div 5k^3h^2 = 3k^4h$. Similarly, $42m^4n^3 \div 7m^3n = 6mn^2$, and $14a^3b^5 \div 2a^3b^2 = 7b^3$. How?

EXERCISES

- | | |
|----------------------------------|-----------------------------------|
| 1. $15a^5b^6 \div 5$ | 11. $27m^4n^5 \div 9mn^2$ |
| 2. $60p^7q^4 \div 12$ | 12. $81r^5h^4 \div 27r^3h^4$ |
| 3. $32mn^5 \div 4n^3$ | 13. $18c^7d^5 \div 9c^3d$ |
| 4. $24r^3d^4 \div 6r^2d^2$ | 14. $36x^4y^5 \div 18xy^3$ |
| 5. $35a^6b^5 \div 7a^4b^3$ | 15. $45r^4s^3 \div 15r^3s$ |
| 6. $75a^5n^6 \div 15$ | 16. $64m^5n^3 \div 16m^5n^2$ |
| 7. $60n^2m^4 \div 4n^2$ | 17. $28w^5v^3 \div 7wv^2$ |
| 8. $56f^2g^3 \div 8fg^2$ | 18. $48b^4q^7 \div 6b^4q^7$ |
| 9. $34a^5y^3z^2 \div 17a^3y^2z$ | 19. $72c^5d^6e^3 \div 9c^3d^2e^3$ |
| 10. $108x^5y^8z^9 \div 12x^5y^3$ | 20. $64m^3n^2k^9 \div 4m^3n^2k^9$ |

As a fraction is an indicated division, simplify the following:

- | | | |
|------------------------------|--------------------------------------|---------------------------------|
| 21. $\frac{35a^3v^2}{7}$ | 25. $\frac{42m^5n^7}{7m^3n^6}$ | 29. $\frac{72a^4b^6}{36ab^4}$ |
| 22. $\frac{27t^5r^6}{9tr^3}$ | 26. $\frac{56m^3n^7k^7}{7mn^3k^6}$ | 30. $\frac{34x^7y^9}{17x^4y^6}$ |
| 23. $\frac{14h^3p^6}{7h^3p}$ | 27. $\frac{48r^5y^3}{12r^3y^3}$ | 31. $\frac{52cd^7}{13cd^5}$ |
| 24. $\frac{38q^7}{19q}$ | 28. $\frac{63s^7v^3y^5}{9s^3v^3y^5}$ | 32. $\frac{128g^3h^4}{32g^2h}$ |

33. Play a number game, using examples in literal numbers.

112. Parentheses.—Parentheses arise frequently in literal numbers. Their meaning is just the same as given on page 24 for Arabic numbers. Review page 24 and solve the exercises there.

As $5(4 + 7)$ means the sum of 4 and 7 multiplied by 5, so $3(a + 2b)$ means that the sum of a and $2b$ is to be multiplied by 3.

EXERCISES

Find the numerical value of the following when $a = 1$, $b = 2$, $c = 5$, $d = 3$, $e = 6$:

- | | | |
|-----------------|-------------------|---------------------|
| 1. $2(a + b)$ | 4. $7(b^2 - a^2)$ | 7. $5(c^2 - b^2)$ |
| 2. $5(2b + c)$ | 5. $3(e + 5d)$ | 8. $3(e^2 - d^3)$ |
| 3. $7(5c - 7d)$ | 6. $4(c^2 + a^2)$ | 9. $7(2b^2 - 8a^3)$ |

10. It has been computed that the distance in feet, D , which an object will fall during any particular second, t , is

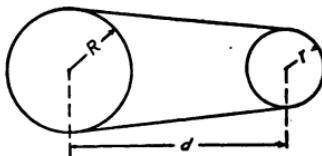
$$D = 16(2t - 1).$$

How far will a ball fall in the 3rd second? in the 5th second? in the 10th second? in the 12th second?

11. The length for the belt connecting two unequal pulleys is given by the equation,

$$L = 3.25(R + r) + 2d.$$

Find the length of belt needed to connect two pulleys whose axles are 14 ft. apart and whose radii are 1.5 ft. and 1 ft.; whose axles are 18 ft. apart and whose radii are 8 in., and 6 in.



12. The length of the belt connecting two equal pulleys is

$$L = 2(\frac{2}{7}R + d).$$

Find the length of belt necessary to connect two pulleys placed 23 ft. apart whose radii are 4 in.

113. Equations with Parentheses.— $5(6 + 3)$ indicates that the sum of 6 and 3 is to be multiplied by 5; or 6 and 3 are each multiplied by 5 and these two products added. Carry out these computations and also for $7(5 + 8)$.

In solving such an equation as,

$$5h = 3(h + 8), \quad (1)$$

the parentheses must first be removed. As we cannot combine h and 8 into one term we make use of the above principle and write the equation,

$$5h = 3h + 24. \quad (2)$$

Find the value of h and test by substituting it in (1).

Similarly,	$5(Q - 4) = 3(Q + 3)$	(3)
hence,	$5Q - 20 = 3Q + 9$	(4) How?
and	$2Q = 29$	(5) How?
	$Q = 14\frac{1}{2}$	(6) How?

Check by substitution in (3).

EXERCISES

Solve the following equations for the literal number and check:

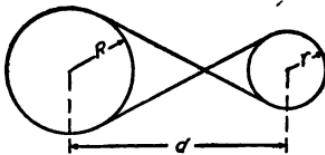
- | | |
|-------------------------|---------------------------|
| 1. $13K = 5K + 18$ | 11. $5R + 3(R - 7) = .11$ |
| 2. $5T - 3 = 2T + 15$ | 12. $6(F - 5) = 2(F + 3)$ |
| 3. $5G = 3(G + 4)$ | 13. $5(B - 3) = 3(B + 3)$ |
| 4. $5(H + 4) = 30$ | 14. $4(G + 3) = 3(7 + G)$ |
| 5. $6(A + 2) = 36$ | 15. $5D - 14 = 3(D - 2)$ |
| 6. $3(M - 2) = 6$ | 16. $7(B - 2) = 5(B + 7)$ |
| 7. $12H = 4(H + 6)$ | 17. $2(Y - 3) = 9 - Y$ |
| 8. $3B = 5(8 - B)$ | 18. $5(C - 3) = 2C - 7$ |
| 9. $5(R + 3) = 2R + 21$ | 19. $2(A - 5) = 3(2 - A)$ |
| 10. $3(S - 2) = S + 46$ | 20. $8(Q - 3) = 5(2 + Q)$ |

Write the equations for the following statements, using parentheses when necessary. Solve the equations and test the results:

21. Find a number which if added to 3 and the sum be multiplied by 4, the result will be 28.
22. Find a number such that 5 times the number equals the sum of the number and 6, multiplied by 3.
23. Four times a number equals the sum of the number and 6, multiplied by 2. Find the number.
24. The sum of a number and 5, multiplied by 3 equals 21. Find the number.

25. For belts crossing, as in the picture, the equation for length is

$$L = 3\frac{2}{3}(R + r) + 2d.$$



Find the length of belt needed for a 3 in. and a 7 in. pulley placed 26 ft. apart; for a 4 in. and a 9 in. pulley placed 18 ft. apart.

Most substances expand when heated. The fractional part of its length that a substance expands when heated one degree centigrade is called its **coefficient** of expansion. The equation gives the new length in terms of the old length, the change in temperature, and the coefficient of expansion, C ,

$$L_n = L_o(1 + Ct).$$

26. Read and interpret the literal equation above.
27. The coefficient of expansion of zinc is 0.000027. What is the length of a zinc bar 12 ft. long heated 10° ? 60° ?
28. A 60 ft. iron rail expands 0.0224 ft. when heated 30° . What is the coefficient of expansion of iron?

114. Literal Fractions.—Literal fractions as well as literal whole numbers arise frequently. In articles bought and sold:

$$n = \frac{P}{p} \text{ and } p = \frac{P}{n}. \quad (1) \text{ See p. 84.}$$

In division:

$$d = \frac{D - R}{q} \text{ and } q = \frac{D - R}{d}. \quad (2)$$

Other illustrations of literal fractions are:

$$\frac{h - 3k}{m^2 + 4n}; \frac{5R^3 - 7K^2}{3G^2 + 8J}; \frac{X^2Y^4 + 13M^6}{5H^3 - 23K^2}.$$

EXERCISES

1. Find n from (1) above if $P = 35$ and $p = 7$; $P = 345$ and $p = 26$.
2. Find p from (1) above if $P = 42$ and $n = 6$; $P = 435$ and $n = 35$.
3. Find d from (2) above if $D = 42$, $R = 3$, and $q = 13$; $D = 115$, $R = 7$, and $q = 12$.
4. Find q from (2) above if $D = 109$, $R = 5$, and $d = 13$.
5. The product of two numbers, n and N , is P . State this as an equation.
6. Express by an equation n in terms of N and P . Express by an equation N in terms of n and P .
7. Find n , if $N = 9$ and $P = 63$; $N = 11$ and $P = 374$.
8. Find N , if $n = 7$ and $P = 98$; $n = 9$ and $P = 435$.

115. Reduction of Fractions.—If the reduction of numerical fractions is understood, no difficulty will arise in the reduction of literal fractions. Thus:

$$\frac{3a^2 c^3}{15a^2 b^2 c^3} = \frac{3a^2 c^3}{7b^2}, \quad \frac{3r^2}{27r^2 t^3} = \frac{3r^2}{4t^4}.$$

Reduce all final fractions to their lowest terms.

EXERCISES

1. Review pages 28 and 29. Solve the exercises on page 29. Reduce the following to their lowest terms:

2. $\frac{15b^3c^2}{5b^2c}$	5. $\frac{80q^2r^5}{16q^4r}$	8. $\frac{7a^3b}{28a^4b^6}$	11. $\frac{25}{40m^3r^2}$
3. $\frac{8x^4y^5}{6xy^6}$	6. $\frac{45}{9ay^3}$	9. $\frac{18j^3h^2}{42j^5h}$	12. $\frac{12hr^2}{16r^7}$
4. $\frac{6m^3}{24m^7}$	7. $\frac{25r^2k^3}{30r^2}$	10. $\frac{17m^5n^7}{34}$	13. $\frac{25hs^3}{35s^5}$

116. Addition and Subtraction of Literal Fractions.—

Literal fractions are added and subtracted just as numerical fractions. The fractions are first reduced to equivalent fractions having the same denominator. It often happens that the numerators cannot be combined into one term, as in numerical fractions. The operations are then merely indicated as in the following:

$$\frac{3a^2}{2b} + \frac{5m}{3k} - \frac{7n}{6bk} = \frac{9a^2k}{6bk} + \frac{10mb}{6bk} - \frac{7n}{6bk} = \frac{9a^2k + 10mb - 7n}{6bk}.$$

EXERCISES

1. Solve exercises 17 through 33 on page 30.

Carry out the following operations:

2. $\frac{2h}{3k} + \frac{5h}{2k}$	5. $\frac{3v}{2m} - \frac{5w}{3m}$	8. $\frac{5m}{6} + \frac{3m}{4} - \frac{2q}{3}$
3. $\frac{2w}{5y} + \frac{3w}{2y}$	6. $\frac{7c}{6} + \frac{4d}{15}$	9. $\frac{2a}{3b} - \frac{3c}{4b} + \frac{5d}{6b}$
4. $\frac{6q}{5r} - \frac{7r}{2q}$	7. $\frac{2d^2}{3c} - \frac{5b^2}{6c}$	10. $\frac{5m}{2b} + \frac{7n}{3b} - \frac{3h}{6b}$

11. Play a number game, using addition and subtraction of literal fractions.

117. Multiplication and Division of Literal Fractions.—Multiplication and division of literal fractions are carried out just as in numerical fractions. Operations are first indicated and then simplified as far as possible by cancellation. Thus:

$$\begin{aligned} \frac{42wx^2}{55q^3r} \div \frac{21w^3xy}{44qs^2} \times \frac{15qy^3}{4wx^2} &= \frac{2}{\cancel{5}\cancel{q}^2r} \times \frac{4}{\cancel{21}w^3xy} \times \frac{3y^2}{\cancel{4}wx^2} \\ &\quad \cancel{pq} \\ &= \frac{6s^2y^2}{qrw^3x}. \end{aligned}$$

EXERCISES

1. Solve the exercises on pages 35 and 38.

Carry out the following operations:

- | | |
|--|---|
| 2. $\frac{2m^3}{3} \times \frac{9}{16m}$ | 9. $\frac{25a^5}{12b^3} \div \frac{10c^4}{34h} \times \frac{8b^2c}{17a^3}$ |
| 3. $\frac{25d^3}{4p^2} \times \frac{12df^3}{25d^5}$ | 10. $\frac{5a}{8b^3} \times \frac{6b^2}{35c^2} \div \frac{10a^4}{7c^2}$ |
| 4. $\frac{36m^3}{25n^2} \div \frac{9m^2}{75n^4}$ | 11. $\frac{27a^3b^3}{8b^5d} \div \frac{28nc^4}{15an^3} \times \frac{20d^2}{7n^3}$ |
| 5. $\frac{20a^4}{39b^3c} \div \frac{45a^6}{27ac^3}$ | 12. $\frac{33x^3}{20r^3} \div \frac{21y^2}{55r^2} \times \frac{12y}{121x^3}$ |
| 6. $\frac{169ab^2}{65m^3} \times \frac{20m^5b}{a}$ | 13. $\frac{36q^2r}{49s^4} \times \frac{28sr}{45q^2} \div \frac{12r^3}{35s^3}$ |
| 7. $\frac{42t^3}{9r^5} \div \frac{35t}{12p}$ | 14. $\frac{15a^5}{14r^2} \div \frac{25}{21r^2b} \times \frac{20rb^3}{9a^2}$ |
| 8. $\frac{13a^7r^3}{28cr^3} \times \frac{25c^3r}{26a}$ | 15. $\frac{18q^3}{25r^7} \times \frac{10r^3}{3q^5} \div \frac{12r}{5q}$ |
| 16. $6t - 5 = 3t + 7$. Find the value of t . | |
| 17. $8r + 4 = 36 - 2r$. Find the value of r . | |
| 18. $7d - 5 = 15 - 2d$. Find the value of d . | |
| 19. $6j + 13 = 48 - 5j$. Find the value of j . | |
| 20. $7r - 9 = 12 - 4r$. Find the value of r . | |
| 21. $5(q - 4) = 30 - 15q$. Find the value of q . | |

118. Fractional Equations.—The solution of an equation often results in a literal fraction. Thus:

$$D = \frac{1}{2}gt^2 \quad (1)$$

expresses the value of D in terms of g and t . g can be found in terms of D and t^2 by writing it in the form:

$$g = \frac{2D}{t^2} \quad (2)$$

which results from dividing both sides of (1) by $\frac{t^2}{2}$.

The value of a literal number in the denominator of a fraction in an equation may be wanted, as a in

$$\frac{3}{2} - \frac{4}{3a} = \frac{5}{2a} - \frac{7}{3}. \quad (3)$$

Multiply each member of (3) by $6a$ when

$$9a - 8 = 15 - 14a. \text{ How?} \quad (4)$$

Adding 8 and $14a$ to both members of (4) gives

$$23a = 23 \quad \text{How?} \quad (5)$$

$$\text{or} \quad a = 1. \quad \text{How?} \quad (6)$$

Test by substituting 1 for a in (3).

Reread pages 87-89. Check all results.

EXERCISES

- | | |
|--|--|
| 1. $9k - 5 = 31 - 3k$; find k . | 9. $\frac{4}{5H} - \frac{2}{3H} = \frac{1}{30}$; find H . |
| 2. $12r - 9 = 8r + 7$; find r . | 10. $\frac{2}{3m} + \frac{4}{5m} = \frac{11}{15}$; find m . |
| 3. $4ak = 16b$; find k . | 11. $\frac{5}{6w} = \frac{1}{4}$; find w . |
| 4. $3mg = 12hk$; find g . | 12. $\frac{3}{v} + \frac{2}{v} = \frac{5}{6}$; find v . |
| 5. $6xy = 15z$; find y . | 13. $\frac{1}{q} + \frac{1}{4} = \frac{3}{2q}$; find q . |
| 6. $6xy = 15z$; find x . | |
| 7. $\frac{1}{3} + \frac{3}{a} = \frac{5}{6}$; find a . | |
| 8. $\frac{3}{4d} - \frac{1}{2} = \frac{1}{4}$; find d . | |

EXERCISES

1. Set up an equation stating that in a rectangular solid length times width times height equals volume.
2. Solve this equation for L in terms of the other numbers. Solve similarly for H . Solve similarly for W .
3. Find the width if the length is 12 ft., the height 8 ft., and the volume 960 cu. ft.

The distance a train or an automobile goes at uniform velocity for any length of time is

$$D = vt.$$

Equations stating a law or a rule are also called **formulas**.

4. Find the velocity in terms of distance and time. Find the time in terms of distance and velocity.

If the weights W_1 and W_2 are placed at distances d_1 and d_2 from a point of support, then from experiment they are found to balance if

$$W_1 d_1 = W_2 d_2.$$



5. Find the value of each number in terms of the others.
6. Where from the point of support must a weight of 5 lb. be placed to balance 10 lb. placed 3 ft. therefrom?
7. 14 oz. placed 6 in. from point of support balances what weight placed 8 in. therefrom.
8. Try this experiment with weights and a yard stick.
9. John, who weighs 60 lb., sits on a teeter-board 7 ft. from the point of support. Where must Harry, who weighs 85 lb., sit so that they balance?
10. James weighs 40 lb. He sits at one end of a teeter-board 14 ft. long. If James is 8 ft. from the point of support, what is the weight of George, who sits at the other end and just balances?

The equation stating the velocity of sound in meters through the air at different temperatures is

$$V = 332 + 0.6t,$$

where t is the temperature on the Centigrade thermometer.

11. How fast will sound travel in one second through air when the temperature is 10° ? 25° ? 32° ?
12. At what temperature will sound travel 344 m. per second? 350 m. per second? 341 m. per second?
13. Change the above equation so as to give the velocity of sound through air in feet per second.
14. What is the velocity of sound through air in feet per second when the temperature is 6° ? 15° ? 20° ?
15. At what temperature will sound travel through air 1,100 ft. per second? 1,200 ft. per second?
16. Solve the above equation for t in terms of V .

The interest of P dollars loaned for T years at $R\%$ is

$$I = \frac{PTR}{100}.$$

17. Explain the last equation.
18. Find I when $P = \$1,500$, $T = 3$ yr., $R = 5$; $P = \$450$, $T = 5$ yr., $R = 7$.
19. Find P when $I = \$360$, $T = 2$ yr., $R = 6$; $I = \$245$, $T = 4$ yr., $R = 8$.
20. Find T when $P = \$2,000$, $I = \$300$, $R = 5$; $P = \$750$, $I = \$150$, $R = 8$.
21. Find R when $P = \$2,000$, $I = \$280$, $T = 2$ yr.; $P = \$850$, $I = \$229.50$, $R = 9$.
22. Solve the last equation for P in terms of I , T , and R .
23. Solve the last equation for T in terms of I , P , and R .
24. Solve the last equation for R in terms of I , P , and T .

VII

PLANE GEOMETRICAL FIGURES



119. Straight Lines.—A carpenter tests a line, as the edge of a ruler, to see if it is **straight** by looking along it to find if there are any humps or depressions. If the edges of two rulers are together at all points as one ruler is slid over the other, they are both straight lines. Why is this so? After this by the word **line** we shall mean a straight line. A straight line will be referred to, as in the picture, by mentioning any two points upon it, as *A* and *B*. This is read “*A-B*.”

120. Parallel Lines.—Two or more lines which have the same direction are said to be **parallel**. The opposite edges of your book, the opposite edges of a door, and the rails of a railroad track are illustrations of parallel lines.

121. Planes.—A smooth flat surface, as the top of your desk, a wall, or a pane of glass, is called a **plane surface** or merely a **plane**. If any two points of a straight line are placed upon a surface and all of the points of the straight line will always lie upon the surface, then the surface is a plane. This is a simple way to test if a surface is a plane. In the picture on page 108 the pupils are making such tests.

122. Plane Figures.—Figures drawn upon a plane, as upon the blackboard or upon the pages of a book, are called **plane figures**. Figures drawn upon a tomato-can or upon any other curved surface are not plane figures. We shall here study only such plane figures as consist of straight lines, circles, or parts of circles.

EXERCISES

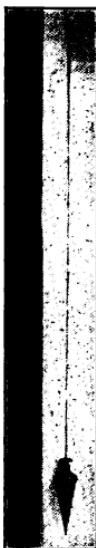
1. Test the following to see which are straight lines: your ruler; the yard stick; the edges of some of your books; the edges of your desk.
2. Slide your ruler along the yard stick or along some other ruler to see if your ruler is straight.
3. Test two or three other lines to see if they are straight.
4. Use one of the lines which you have found to be straight, or a string drawn out, to test which of the following are planes; the top of your desk; the floor; the covers of some of your books; the door; the radiator.
5. Suggest some other object and test its surface as a plane.
6. Give three illustrations of parallel lines which are in your schoolroom; outside of your schoolroom.

123. Vertical Lines.—If some heavy object, as a bolt or a nail, is tied to one end of a string while the other end is held firmly, the string will point toward the centre of the earth. The string is said to be a **vertical line**. The whole apparatus is called a **plumb-line**, and the weight is called the **plumb-bob**. The picture shows a plumb-line used by surveyors and builders. While all plumb-lines point toward the centre of the earth those that are near together are so nearly parallel that they are called parallel.

124. Horizontal Planes.—A **horizontal plane**, also called a **level plane**, is one like the floor, the ceiling of a room, or a table which does not slope. The tops of all walls in the foundation of a building must be level or the floors will slope. Builders continually test their work with an instrument called a **spirit-level**. This consists of a small glass tube which is sealed at both ends and slightly curved upward in the middle. It is filled with spirit liquid except for a small bubble of air. Why are spirits used in the place of water? The air bubble will always be at the highest point of the tube. If *A* and *B* are both equally high the air bubble will be at *C*, the mid-point of the tube. If *A* is higher than *C* then the air bubble will be on the side of *C* toward *A*. Where will the bubble be if *B* is higher than *C*?

A level can easily be made from a small glass vial filled with water, except for a small bubble of air.

In order to test a plane, as the top of your desk, to see if it is a level plane, levels must be taken in two directions as shown in the picture on the next page. Why is it necessary to take two levels?





125. Vertical Planes.—Planes which are always parallel to a plumb-line hung near them are called **vertical planes**. Such planes are the walls of a room.

EXERCISES

1. Make a plumb-line, as suggested, and hold it up to get the idea of a vertical line.
2. Suggest two or three lines in the room which you think are vertical lines. Test them.
3. Hang up two plumb-lines about a foot apart. Measure the distance between the two lines at several places. What do you find? What do the other pupils find? What do you think will happen if you hang the two plumb-lines together in still a different place?
4. Test the following to see which are vertical planes: a wall; a window-pane; a side of the teacher's desk.
5. Make a vial-level and test the following planes to see which are horizontal: the floor; a window-ledge; the top of your desk.
6. If your desk is not level, place a book upon it and raise the lower edge of the book until it is level. Now test the book again to see if it is level throughout.
7. Harry's mother finds that the bottom of her oven slants. How can Harry find which way it slants? How can the oven be levelled? Make such a test of an oven at the school or at home.
8. Play a number game, using exercises in fractions.



126. Straight Line Measurements.—One of the most common measurements is finding the distance between two points on a straight line. For long distances surveyors use a chain 66 ft., or 100 ft. long. Distances in a room are usually measured in feet or meters. Smaller distances, as the length or width of your book, are measured in inches or centimeters and fractions of inches or centimeters. Very minute distances, as diameters of very thin wires, are found by extremely delicate instruments, **vernier calipers** or **micrometers**. These delicate instruments we shall not use.

127. Measuring With the Ruler.—In measuring the distance between two points, place the ruler as shown for the upper line, so that the measuring line is right against the line to be measured. Do not use the other form. Why? Begin measuring from some point other than the end of the ruler because the end is often worn, hence not accurate.



128. Stepping Off Distances.—Such distances as the width of a street are needed only to within one foot. What does this mean? It is usual to find such lengths by counting the number of steps they are long. Then multiply this by the length of one step.

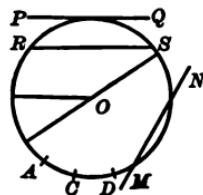
129. Drawing Lines.—In order to join two points by a straight line first place the pencil upon one point, and the ruler against it. Now with the left hand push the ruler from you, as in the picture, until the straight edge falls upon the other point also. Hold the ruler firmly by the left hand while drawing the line.



EXERCISES

1. Estimate the height of the teacher's desk. Measure it. What is your error? Your error is what per cent of the height? It is usual to call this the "per cent of error."
 2. Make the same measurements and estimates, and find the per cent of error for: width of the door; width of a window; width of your desk; length of the teacher's desk.
 3. Draw upon the blackboard, without using your ruler, a horizontal line that you think is 1 ft. long; 3 ft. long. Find your per cent of error.
 4. Repeat Ex. 3, drawing vertical lines.
 5. Draw a line and measure it as accurately as possible in both inches and centimeters.
 6. Draw a line 10 in. long. Find its length in centimeters. From this compute the length of 1 in. in centimeters.
- Find the following distances by stepping them off:
7. The length and the width of the schoolroom.
 8. The width of a street.
 9. The length of a city block.
 10. The length of any other distance after estimating it. Find the per cent of error of your estimate.

130. Circles.—Circles are continuous curved lines, all points of which are equally distant from a point called its **centre**. The distance of any point on the circle from the centre is called the **radius**. Two or more are **radii**. The distance of a point on the circle to an other point on the circle, straight through the centre or twice the radius, is called the **diameter**. The total curved line is called the **circumference**. Any part of the circumference, as ACD , is called an **arc** and is written \widehat{ACD} . A line cutting the circle, as MN , is called a **secant**. A line just touching the circle, as PQ , is called a **tangent**. A line joining two points on the circle, as RS , is called a **chord**.



Circles are drawn upon the board by means of a piece of chalk fastened to a string. How? This idea can be used to draw circles upon the ground. How? A compass is used to draw circles upon paper as is shown below. How is the compass held in the picture? Always hold a compass thus. Why?

131. Segments of a Line.—A part of any line is called a **segment**.

The compass is often used to cut off segments of lines. For instance, it may be required to lay off on AB , from A , a line equal in length to some given line, MN . The compass is spread out with one point placed upon M and the other upon N . One point of the compass is now placed at A and an arc struck cutting the line at C . AC is then equal to MN . Why?



EXERCISES

1. Upon paper or the board construct a circle of any radius. Point out its centre; radius; diameter; circumference.
2. Locate any point on your paper and with this point as centre draw a circle whose radius is 1.25 in.; 3.5 cm.
3. Mark two points upon one of these circles and join them by a chord.
4. Express by an equation that the diameter equals twice the radius. Solve the equation for r .
5. Find the radius of the circle whose diameter is 4 ft.; 6.4 dm.; 11.5 ft.; 7.3 m.
6. Construct a circle with any point as centre, having a diameter of $1\frac{1}{2}$ in.; 3 cm.
7. Construct a circle with a radius of 2 in. Mark a point upon the circle and draw a tangent to the circle at this point.
8. Draw a line of any length. Set the compass to measure 2 in. and cut this off from the line.
9. Draw a circle with radius 2 in. and mark a point, Q , upon it. Draw two chords through Q , each 3 in. long. Use the compass to find the points which are to be joined to Q .
10. Mark two other points upon the same circle and join them by a secant.
11. What is the longest chord that can be drawn in a circle? What is its length for each circle constructed for Ex. 2?
12. Construct any circle and mark a point within it. Through the point construct the longest possible chord.
13. Play a number game, using addition of decimals.

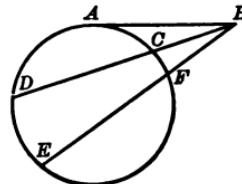
132. Drawing a Tangent to a Circle.—To draw a tangent to a circle through a point without the circle, place the pencil on the point and push the ruler from you against the pencil. Now rotate the ruler about the pencil until it touches the circle. Hold the ruler firmly with the left hand and draw the tangent.



EXERCISES

1. Construct a circle of radius $1\frac{1}{2}$ in. Locate a point outside the circle. How many lines can be drawn through this point tangent to the circle? Draw them.

2. Draw a circle of any radius. Draw a tangent and two secants as in the figure. Measure AB very accurately and square the number. Measure BC and BD and find the product of these numbers. Do the same for BF and BE . Compare the three results. What did the other pupils find? What is your conclusion?



3. It is shown in geometry that the square of the length of a tangent, as AB , always just equals the product of the two segments of a secant from a point on the tangent, as $BD \times BC$. State this as an equation.

4. If AB is 12 in. and BD is 16 in., how long is BC ?

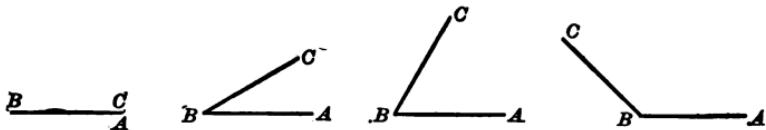
5. Circles having the same centre are called **concentric**. Draw two concentric circles with different radii. From points on the larger circle draw three tangents to the inner circle. Measure these lengths. What do you find?

133. Proofs.—Much emphasis has been placed upon the need of checking all computations. There is another form of proof that will arise often after this which you must learn to use and to understand fully.

A brick or a rock thrown into the air falls to the ground upon any of its sides. A weight fastened to a handle, as a hammer, if thrown into the air falls to the ground so that the weight strikes the ground first. Try this. Suppose this happens with all such objects thrown into the air by you and by all of the pupils of the room, what do you think will happen with any such object thrown into the air by any one at any time? Trials like these are called experiments. You have here shown experimentally that a weight attached to a handle, if thrown into the air, falls so that the weight always strikes the ground first. The more people who have made the trials and the more different trials each has made, the more reliability will be attached to the results obtained. In Exs. 2 and 5, on page 116, similar experiments were carried out. What was proved in each of these exercises?

EXERCISES

1. Place a book upon your desk and a heavier one on top of it. Pull the lower book slowly. What happens? Jerk the lower book quickly. What happens? State what you have proved.
2. Hold a rubber-ball or a tennis-ball 4 ft. above the floor. Drop it and measure the height of the first bound. What fractional part of the distance dropped was the rebound? Repeat for five other distances. What do you find?
3. Drop a small and a large stone at the same time. Do they fall together? Do the same with a stone and a wad of paper. What is your conclusion?



134. Angles.—Suppose that two lines AB and BC coincide, like the two blades of shears, and that while one is held fixed, as AB , the other is rotated about B . The side BC is said to **generate an angle**. The two lines are the **sides** of the angle and their point of intersection the **vertex** of the angle. An angle is designated by naming a point in one side, A , the vertex, B , and a point in the other side, C . This is written **angle ABC** , or $\angle ABC$. It is read “angle A - B - C .”

Draw any three angles and letter each, using letters other than those above. Read each angle. Name the vertex of each angle.

Point out three angles in the room. Which is the largest? Which is the smallest?

135. Measurement of Angles.—The more the moving line is turned, the greater will be the angle. Suppose that BC is turned completely around so as to come again into the position AB . If this has been carried out by 360 equal movements of BC each of these little 360 angles is called a degree. Hence, the sum of all the angles about a point is 360 degrees. Each degree is divided into 60 equal minutes and each minute into 60 equal seconds. An angle of five degrees, nine minutes, eleven seconds is written $5^{\circ} 9' 11''$. Never use the symbols for minutes and seconds of an angle as abbreviations for minutes and seconds of time.

This division of the angles about a point was used by the ancient Babylonians thousands of years ago. They thought that the sun went around the earth once every 360 days, passing through the same angle each day.

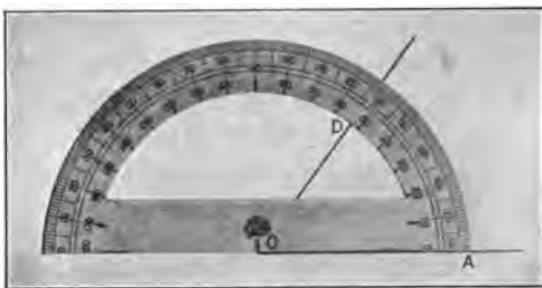
136. Protractor.—An instrument called a protractor is used for measuring angles.

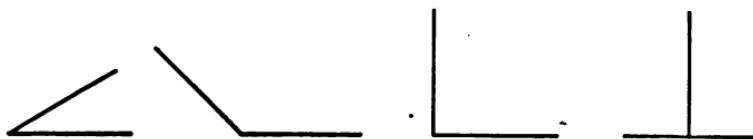
To measure an angle the protractor is placed so that OA falls on one side of the angle with the vertex at O . The place where the other side of the angle crosses the semicircle, as at D , is then read, giving the angle in degrees. If the sides of the angle are shorter than the radius of the protractor, they must first be lengthened. Does the size of the angle depend upon the length of the sides?

Note that $\angle AOD$ equals 53° .

EXERCISES

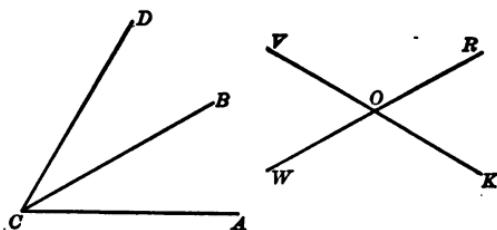
- Arrange the angles at the top of page 118 in the order of their size.
- Draw any three angles and measure them accurately.
- What angle does the minute hand of a clock generate in 15 min.? in 40 min.? in 20 min.? in 10 min.? in 30 min.?
- A wheel has six spokes so set as to form the same angle between any two successive spokes. What is the angle?
- $6 \angle A - 60^\circ = 2 \angle A + 40^\circ$. Find $\angle A$.
- $\frac{2}{3} \angle X + 20^\circ = \frac{1}{4} \angle X + 85^\circ$. Find $\angle X$.
- $\frac{3}{5} \angle K - 36^\circ = 2^\circ - \frac{2}{3} \angle K$. Find $\angle K$.
- Play a number game, using English and metric denominative numbers.





137. Kinds of Angles.—Angles of 90° are called **right angles**, LR , and their sides are said to be **perpendicular**, \perp , to each other. Angles less than a right angle are called **acute angles** and angles greater than a right angle are called **obtuse angles**. Angles whose sides form a straight line are called **straight angles**. They contain 180° . Angles greater than 180° are called **reflex angles**.

138. Relations of Angles.—Two angles which have a side and the vertex in common, as $\angle ACB$ and $\angle BCD$, are **adjacent angles**. Angles whose sides are extensions of the sides of another angle, as $\angle ROK$ and $\angle WOV$, are **vertical angles**. Two angles, M and N , whose sum is 90° , are **complementary angles**, and two, X and Y , whose sum is 180° , are **supplementary angles**. In equations this is



$$\angle M + \angle N = 90^\circ;$$

$$\angle X + \angle Y = 180^\circ.$$

EXERCISES

1. Draw an acute angle; an obtuse angle; a reflex angle.
2. Give three illustrations of perpendicular lines.
3. Test the covers of some of your books to see if the edges form right angles.

4. Use the cover of some book in drawing a right angle. Next extend the sides from the vertex so as to form four angles. Measure the sizes of the other angles. What is your conclusion?

5. Draw two intersecting lines. Pick out the vertical angles. Measure as accurately as possible each pair of vertical angles. What do you find?

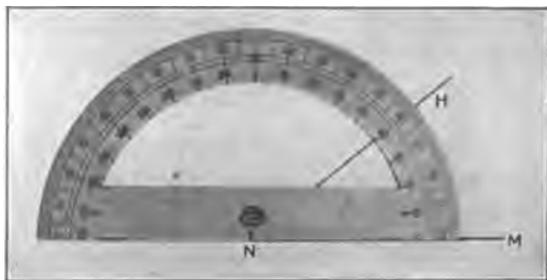
6. Give the complement of 25° ; of $32^\circ 7' 15''$; of $48^\circ 29' 37.7''$.

7. Give the supplement of 45° ; of $138^\circ 9' 27''$; of $65^\circ 38' 12''$.

8. Draw an acute angle. Measure it and find its complement; its supplement.

9. Which of the following are acute, obtuse, reflex: 35° ? 213° ? 165° ? 352° ? 127° ? $33\frac{1}{4}^\circ$? $90^\circ 3'$?

139. Construction of Angles.—An angle of any size can be constructed on any line, NM , as a side with the vertex at any point on it, as N . Place the protractor as shown in the figure. Read the required angle on the semicircle and mark it at H . Join N to H .



The required angle is $\angle MNH$.

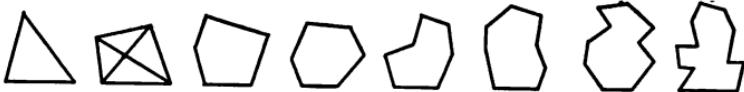
EXERCISES

1. Draw a line and mark a point upon it. Draw a second line through the point making an angle of 30° with the first line. Repeat for angles of 45° ; 60° ; 150° ; 230° .

2. Construct the angle that is the supplement of 135° ; of 83° ; of 95° ; complement of 67° ; of 45° ; of 30° .

140. Straight Line Figures.—Figures made up of straight lines are grouped according to their number of sides. Figures of more than four sides are generally called polygons, many sided figures. Figures whose sides as well as angles are equal are called **regular figures**.

Name	Number of Sides	Name	Number of Sides
Triangle	3	Heptagon	7
Quadrilateral	4	Octagon	8
Pentagon	5	Decagon	10
Hexagon	6	Dodecagon	12



141. Diagonal.—In a plane figure a line joining one vertex to another vertex not on the same side is called a **diagonal**.

142. Perimeter.—The distance around a plane figure is called its **perimeter**.

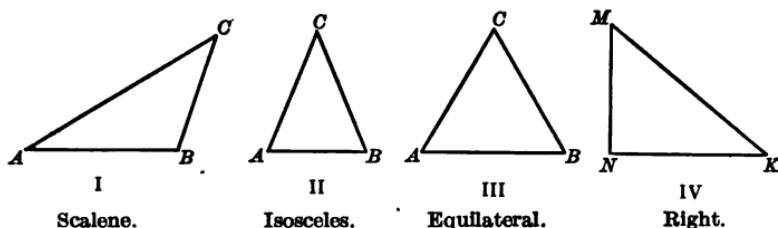
EXERCISES

1. Name each of the figures according to the table above.
2. Draw a quadrilateral, a hexagon, and an octagon.
3. Draw all the diagonals possible for each of the figures in Ex. 2. Count the number of diagonals in each case. If the number of sides is n , the number of diagonals, N , will be

$$N = \frac{n(n - 3)}{2}$$

Check this equation by using the numbers you have found.

4. Use the last equation to find the number of diagonals in a decagon; in a dodecagon.
5. Find the perimeter of your desk; of the schoolroom; of any other object.



143. Triangles.—A triangle is **scalene** if none of its sides are equal, it is **isosceles** if two sides are equal, and it is **equilateral** if all three sides are equal. A triangle containing a right angle is called a **right triangle**. The side opposite the right angle is named **hypotenuse**. The symbols are $\triangle ABC$ and $\triangle MNK$.

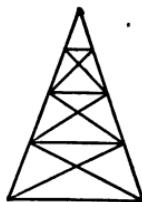
Any side of a triangle is less than the sum of the other two sides, because it is shorter to go from A to B along the straight line AB than from A to C to B . In symbols this is stated,

$$AB < AC + CB. \quad (1)$$

Such a statement is called an **inequality** because it says that two numbers are **not equal**. Toward which number does the vertex of the symbol $<$ point?

EXERCISES

1. Here is a picture of a part of a steel tower used in a manufacturing plant. Find the various kinds of triangles in it.
2. Look for the various kinds of triangles on rugs, wall paper, buildings, or anything else. Tell the class about them to-morrow.
3. The sides of a triangle are $18\frac{3}{4}$ in., $16\frac{7}{8}$ in., and $27\frac{1}{2}$ in. Find its perimeter.
4. Draw two equal lines from any point. Join the other ends of the two lines. What kind of triangle is this?



5. Measure each angle opposite the equal sides. What do you find? What do the other members of the class find? You will prove much more accurately in geometry that the angles opposite the equal sides in an isosceles triangle are equal.

6. Draw any triangle and cut it out from the paper. Next tear off two of the angles and place them together with the other angle. Show from this, that the sum of the angles of a triangle equals 180° , a straight angle. This is also proved much more exactly in geometry.

Find the third angle of a triangle if two angles are:

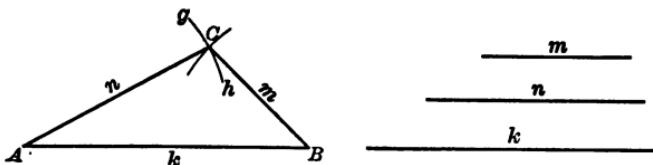
7. 60° and 40° .

8. $32^\circ 15' 23''$ and $48^\circ 56' 18''$.

9. $18^\circ 45' 37''$ and $47^\circ 38' 42''$.

10. The school gardening class in a junior high school has a flower border 42 ft. long, which they wish to place around an equilateral triangle. How long should the sides of the triangle be to use all of the flower border?

144. Construction of a Triangle with Given Sides.—A triangle can be constructed whose sides are of any given lengths as m , n , and k , if no one side is longer than the sum of the other two. Why cannot one side be longer than the sum of the other two? First draw a straight line of any length and then lay off a segment upon it by use of the



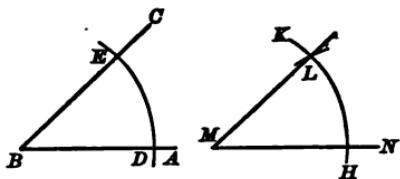
compass, as AB , equal to one of the given lines, as k . Set the compass to measure one of the other two lines, as n .

With this as a radius and A as a centre strike an arc gh . Next set the compass to measure the third side, as m , and with this as a radius strike an arc about B as a centre. Suppose that these two arcs intersect at C . Join C to A and C to B ; this completes the triangle. Study this construction to make sure that it will give the required triangle.

EXERCISES

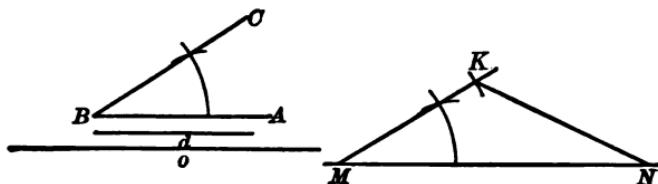
1. Construct the triangles which can be made with the following sets of sides: 6, 8, 10; 9, 4, 6; 11, 5, 4; 9, 3, 12; 8, 6, 12.
2. Repeat any one construction in Ex. 1 and cut out the triangle from the paper. Can you make it fit the first having these same sides?
3. Construct an isosceles triangle with one side 1.5 in. and two sides each 2 in.
4. Construct any other isosceles triangle.
5. Construct an equilateral triangle whose sides are 3 in.
6. Construct any other equilateral triangle.
7. Measure the angles of the triangle constructed in Ex. 6. What do you find? What do the other pupils find? What does this prove?
8. For any isosceles triangle—see page 123—
$$2 \angle A + \angle C = 180^\circ.$$
Solve this for $\angle C$; solve for $\angle A$.
9. If the two equal angles of an isosceles triangle are each $35^\circ 14'$, what is the third angle?
10. One angle of an isosceles triangle is 34° and the other two angles are equal. How many degrees are there in each of the equal angles?
11. What would the above equation be for an equilateral triangle? How many degrees are there in each angle of an equilateral triangle? Would it be correct to call the equilateral triangle an equiangular triangle?

145. Construction of an Angle.—To construct an angle equal to a given angle ABC , having MN as a side with vertex at M , first describe an arc with the compass about B as a centre. Describe an arc about M as a centre with the same radius. Measure with the compass from D to E . Use this as a radius and H as a centre and describe an arc cutting the arc HK in L . Join M to L . $\angle HML$ is the required angle. Notice that $\angle HML$ could be cut out and placed over $\angle ABC$. Why is this so?

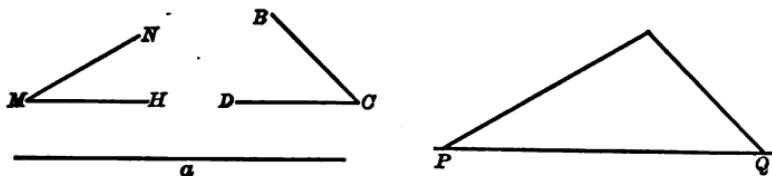


146. Construction of Triangles with Given Sides and Angles.—A triangle can be constructed which shall have an angle equal to a given angle and the two sides of this angle equal to two given sides. First draw a line of any length and lay off upon it a segment MN equal to one of the given sides, as d . Then construct an angle, as in Art. 145, equal to the angle ABC , with one side MN and vertex at M . $\angle NMK = \angle ABC$. Lay off MK equal in length to the other given line, d . Join N to K . the $\triangle NMK$ is the required triangle. Again study the construction of the required triangle.

If a second triangle is constructed with equal sides and included angle; could one be cut out and placed over the other? Why?



A triangle can also be constructed which has a side and the two angles at the ends of the side given. Draw a line

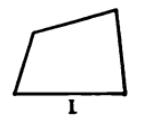


of any length and lay off a segment, as PQ , equal to the given line a . At one extremity, P , construct an angle equal to one of the given angles, $\angle HMN$. At the other end of the line, Q , construct an angle equal to the other given angle, $\angle BCD$. Two sides of the constructed angles will meet in a point. This point is the third vertex of the required triangle.

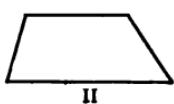
EXERCISES

1. Draw any angle and a line CD . At C construct an angle equal to your given angle and having CD as one side.
2. Construct a triangle having one angle 54° with the sides including it, 3 in. and 2 in.; an angle 104° with the sides including it, 5 in. and $2\frac{1}{2}$ in.
3. Draw any angle and two lines. Construct a triangle having the lines as sides and the angle included.
4. Construct an isosceles triangle having the equal sides 4 in. and the angle between them 68° . What is the size of the equal angles?
5. Draw two angles and a line. Construct a triangle having the two angles and the side between them.
6. Construct a triangle with one side 2.25 in. and the angles including it, 35° and 55° . What is the third angle?
7. Construct a triangle whose sides are 3, 4, and 5 inches; 5, 12, and 13 inches. Measure the angles. What do you find?

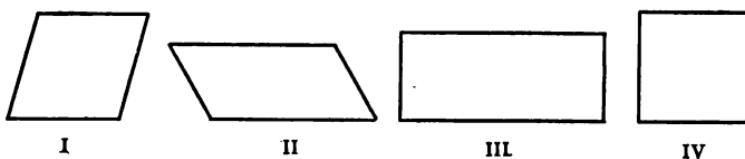
8. Construct a triangle having a side 3 in. and the angles at its ends 40° and 120° ; a side 4 in. and the angles at its ends 100° and 45° .
9. Find the third angle of a triangle if two of its angles are 40° and 56° ; $63^\circ 4' 17''$ and $107^\circ 47' 32''$.
10. Draw any triangle. Find its perimeter.
11. Find the sum of the angles of the triangle used in Ex. 10. If the sum of the angles of a triangle is 180° , what was your error in measuring the angles of this triangle? What was the per cent of error?
12. Construct an isosceles triangle whose equal sides are each 4.5 in. and the third side 3.25 in.
13. Measure any one angle in the triangle constructed in Ex. 12 and from that compute the other angles.
14. Construct a triangle whose sides are 4.5 in., 3.2 in., and 5.6 in. Measure two angles and compute the third angle.
15. A watch keeping accurate time can be used on a sunny day to tell directions. Hold the watch so that the reflection of the sun seen on the face of the watch appears midway between 12 and the place to which the hour hand points. The 12 will then be south. Try this experiment.
16. When the 12 of a watch is held due south, where will the reflection of the sun be seen at 9 A.M.? noon? 2 P.M.?
147. **Quadrilaterals.**—A quadrilateral having no sides parallel is called a **trapezium** (I); having only two sides parallel is called a **trapezoid** (II); having two sets of parallel sides (top page 129) is called a **parallelogram**. A parallelogram with unequal sides and angles not right angles is called a **rhomboid**; with equal sides but angles not right angles is called a **rhom-**



I



II

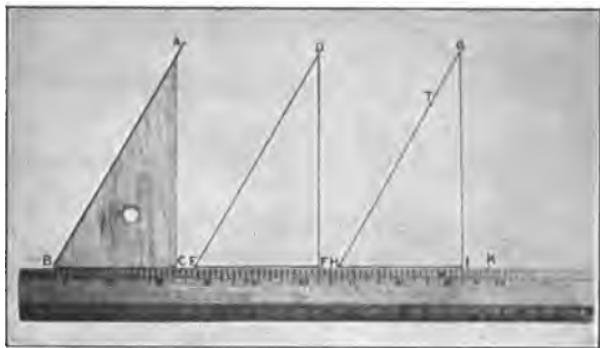


bus; with unequal sides but angles right angles is called a **rectangle**; with sides equal and angles right angles is called a **square**.

In any parallelogram, opposite sides are equal; opposite angles are equal; a diagonal divides it into two equal triangles; each diagonal bisects the other.

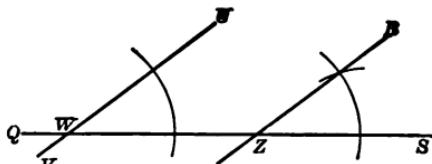
EXERCISES

1. Draw free hand and label each kind of quadrilateral mentioned in Art. 147. Note differences closely.
2. Draw any quadrilateral and one diagonal. Note that the sums of the angles of the quadrilateral equals the sum of the angles of the triangles into which the diagonal divides the quadrilateral. What is the sum of the angles of a triangle? Hence what is the sum of the angles of any quadrilateral?
3. Find the fourth angle in a quadrilateral which has three angles, $46^\circ 14' 35''$, $135^\circ 34' 17''$, $97^\circ 45' 16''$
4. Find the fourth angle in a quadrilateral which has the three angles: $46^\circ 38' 47''$, $78^\circ 19' 17''$, $109^\circ 35' 29''$.
5. One angle of a parallelogram is $78^\circ 45' 16''$. What are the other 3 angles?
6. One side of a rhombus is 6.75 in. What is its perimeter?
7. Play a number game on finding angles, complements of angles, and supplements of angles.



148. Construction of Parallel Lines.—A draftsman would draw a line through a point T , parallel to a line AB , by first placing a triangle in the position ABC . He would then place a straight edge along one side of the triangle, as BH , and slide the triangle along BH until the side AB falls upon the point T . Why will a line then drawn along the side of the triangle through T be parallel to AB ?

Note that the parallel lines AB , DE , and GH cut HB at the same angle. This is used in drawing a line through a point, Z , parallel to a line, UV . First draw any line through Z , as QZ , cutting UV in W . Next construct a line through Z making the angle SZR equal to the angle ZWU . Line ZR is parallel to UV . Why?

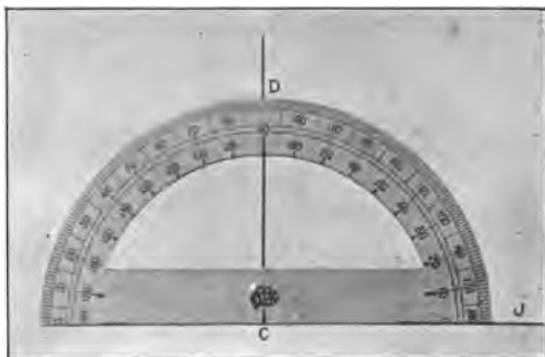


EXERCISES

1. Draw any line and mark a point outside. Through the point construct a line parallel to the first line.
2. Draw two intersecting lines. Place a point upon each line. Through these points construct lines parallel to the opposite lines. What quadrilateral is this?

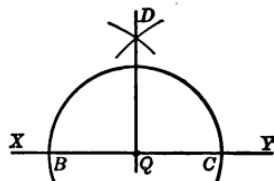
149. Drawing Perpendiculars.—When are lines perpendicular? Suppose that triangle ABC is pushed along until AC falls upon point T . A line now drawn along AC will pass through T and be perpendicular to HB . Why? How can a line be drawn through K perpendicular to HB ?

To draw a line through C perpendicular to CJ place a protractor as in the figure and mark D at the point 90° .



Join D to C . $\angle JCD = ?$ Why is $CJ \perp CD$?

Here is a plan much used by architects and draftsmen. To construct a line through Q perpendicular to XY , first draw a semicircle about Q , with any radius. With a radius longer than BQ draw arcs using B and C as centres. Join their point of intersection, D , to Q , hence $DQ \perp BC$.



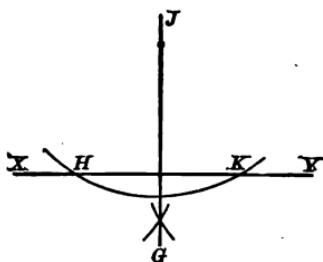
Joining B and C to D forms two equal triangles. Why? Then $\angle DQC = \angle DQB$. Why? How many degrees are there in each of these two angles? Why is $BC \perp DQ$?

EXERCISES

Draw any straight line. Through any point on the line construct a perpendicular by use of—

1. The right triangle.
2. The protractor.
3. The compass.

150. Perpendicular Through Point Outside Line.—One method, by moving a triangle, has already been given—page 131. Here is another: With J as centre, construct an arc cutting XY in H and K . With these points as centres draw arcs having a radius longer than $\frac{1}{2} HK$. Join their point of intersection, G , to J . The line JG is perpendicular to the line XY . Test by measuring the angles between the two lines.



151. Distance of a Point from a Line.—The length of the perpendicular from a point to a line is called the **distance of the point from the line**. How far is J in the above figure from the line XY ?

EXERCISES

1. Draw any line and mark a point outside of the line. Construct a line through the point perpendicular to the line. Use the two methods given.
2. Draw any line and mark a point outside of the line. Find the distance that the point is from the line.
3. Draw a line of any length and construct a perpendicular at its mid-point. Select any point in this perpendicular and measure its distance from each end of the line. What do you find? Repeat for another point on the perpendicular. What do you find? What does this prove? Fold the paper about the perpendicular. What does this show?
4. Draw any triangle. Construct perpendiculars to each side at its mid-point. What do you find? What did the other pupils find? What does this show?

5. Measure the distance of the point of intersection of the perpendiculars from each vertex. What do you and the other pupils find? What does this prove?

Draw a circle with the point of intersection of the perpendiculars as centre and its distance from each vertex as the radius. The circle passes through the vertices and is said to **circumscribe the triangle**.

6. Draw a triangle and circumscribe a circle about it.

7. Mark any three points upon the paper and draw a circle passing through them. See Ex. 5.

8. Draw any triangle. Draw the lines bisecting each angle. What do you find? What did the other pupils find? What does this show?

9. Find the distance of the intersection of the bisectors from each side of the triangle in Ex. 8. How do you find the distance of a point from a line?

10. With the point found in Ex. 8 as centre and the distance of this point from each side of the triangle as radius, draw a circle. This circle will be tangent to each side of the triangle and is said to be **inscribed in the triangle**.

11. Draw any triangle and inscribe a circle in it.

12. Draw any triangle. From each vertex construct a line perpendicular to the opposite side. What do you find?

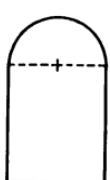
13. Draw any triangle. Join each vertex with the midpoint of the opposite side. What do you and the other pupils find?

14. Draw two lines, not too long, intersecting each other. At the extremity of each construct a line parallel to the other line. What figure does this form? How should the lengths of opposite sides compare? Measure these to check your work.

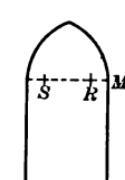
15. Construct a parallelogram with sides 1.8 in. and 2.25 in. and the angle between them 60° . Check the work by measuring each pair of opposite sides.



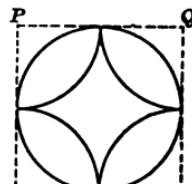
Cathedral of Amiens.



Roman.



Gothic.



Circular windows.

16. Construct any parallelogram. Draw the diagonals. Do the diagonals bisect each other? What do the other pupils find? What is the conclusion?

17. Construct a rectangle whose sides are $2\frac{3}{4}$ in. and $1\frac{7}{8}$ in. Measure each diagonal. What do you find?

18. Construct a rectangle with sides of any length. Measure its diagonals. What do you and the other pupils find? What is the conclusion?

19. Construct a square whose sides are 2.75 in.

20. Construct any square and inscribe a circle in it. Circumscribe a circle about the square.

21. Pick out from the picture on the opposite page a circle, and as many as you can of the various kinds of angles, triangles, and polygons that you have studied.

22. Look up some interesting facts about the Cathedral of Amiens. What type of architecture is illustrated in this cathedral?

23. Construct a pattern of the Roman window in Fig. I. Make the rectangular part twice as long as it is wide.

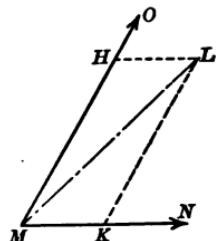
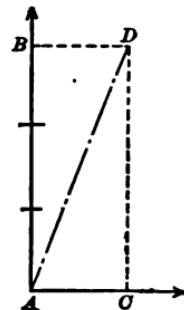
24. Construct a pattern of the Gothic window in Fig. II. Make MN three times as long as NH . To make the arch use R and S as centres and MS as a radius.

25. Construct a pattern of the circular window in Fig. III. Use the square $PQSR$ and finally erase these lines.

26. Play a number game, using fractions.

152. Combining Motions.—If you were on a ship which was sailing north while you were walking east on the deck, you would be going neither north nor east but in a direction between north and east. If you were walking 3 ft. per second on a ship at rest, you would go from A to C in 1 sec. The ship sailing 9 ft. per second would carry you from A to B in 1 sec. if you were not walking. Your actual motion walking on the deck of the moving ship would be from A to D in 1 sec. The direction of AD will be the direction of your actual motion and its length will tell the number of feet per second of your motion.

Suppose that you were on a ship going 7 ft. per second along MO and that you were walking along the deck 3 ft. per second along MN . Your motion due to the ship would be represented by MH , which is 7 spaces. Your motion of walking would be represented by the 3 spaces MK . To find your actual motion, through H draw a parallel to MN and through K draw a parallel to MO . Your actual motion will be the direction ML and an amount in feet per second equal to the number of divisions ML is long. How long is this?



EXERCISES

1. Draw the lines AB and AC above, making the divisions $\frac{1}{4}$ in. long. What angle does the direction of motion AD make with the motion of the ship AB ? How many quarter inches is AD ? How many feet per second is the motion from A to D ?

2. Make the drawing for the second figure on page 136. Let each space on the two lines be $\frac{1}{4}$ in. Let the angle between MN and MO be 60° . Find the angle that the direction of motion, ML , makes with the direction of walking, MN . Find also the angle that the direction of motion makes with the direction of motion of the ship MO . What is the sum of these two angles? Did you make any error in measuring them? What was your per cent of error? Measure the length of ML to find the number of feet per second of the actual motion.

3. Find the three motions for Ex. 1 on page 136 in miles per hour, in the place of feet per second.

4. Find the three motions in Ex. 2 above in miles per hour, in the place of feet per second.

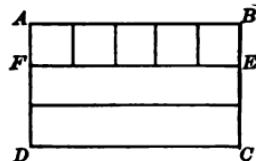
5. A train is going 12 mi. per hour and a boy is walking along one of its flat-cars at the rate of 3.5 ft. per second in a direction making an angle of 40° with the motion of the train. Represent the motion of each and find the resulting motion. What is the direction of the resulting motion? How many feet is it per second? miles per hour?

6. A sailor climbs straight up on a mast at the rate of 4.5 ft. per second, on a ship going at the rate of 9 mi. per hour. Represent each motion and the combined motion of the sailor. How many feet per second does the sailor really move? What angle does his motion make with the deck of the ship?

7. A boat is rowed at the rate of 1.5 ft. per second at right angles to a stream flowing 3 ft. per second. Find the actual motion of the boat and its direction.

8. A balloon is rising directly upward 5 ft. per second. A wind is carrying it off 18 ft. per second. Find the actual motion of the balloon.

153. Areas of Rectangles.—Divide the rectangle $ABCD$ into strips each a unit wide, as an inch or a foot. There will be as many strips like $ABEF$ as the rectangle is units wide. Each strip will contain as many square units of area, square inches or square feet, as the rectangle is units long. The total area will therefore equal the number of units of area in one strip, which is the same as the number of units of length, times the number of units of width. This is stated simply by saying that the area of a rectangle equals its **length times its width**, that is,

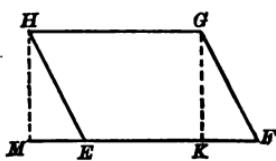


$$A = wl. \quad (1)$$

As the vertical dimension is called the height or altitude and the horizontal dimension is called the base, this formula is often written,

$$A = bh. \quad (2)$$

154. Areas of Parallelograms.—Draw the line GK in the parallelogram, $FGHE$, perpendicular to FE . If now the part FGK is cut off it can be placed in the position EHM . F goes to E , G to H , and K to M . The figure $GHMK$ will be a rectangle. The perpendicular distance between either pair of parallel sides, as GK , is the altitude of the parallelogram. Show that the parallelogram and the rectangle have equal altitudes; also equal bases. Hence, as they have equal areas the formula for the area of any parallelogram is the same as for a rectangle,



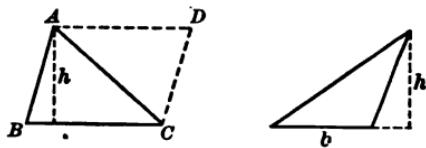
$$A = bh. \quad (3)$$

EXERCISES

1. Find the area of the top of the teacher's desk.
2. Find to two decimal places, the area of the rectangles which are 5.25 in. by 6.75 in.; 6.34 cm. by 8.4 cm.
Always use abbreviated multiplications and divisions with decimals.
3. What is the perimeter of each rectangle?
4. In a well-lighted schoolroom the window space is at least 15 % the area of the floor. Test your schoolroom.
5. Find the second dimension in a rectangle whose area is 352.6 sq. ft., if the base is 28 ft.
6. From equation (1) on page 138, find l in terms of A and w ; find w in terms of A and l . From (2) and (3) find b in terms of A and h ; find h in terms of A and b .
7. Find to two decimal places, the second dimension of a rectangle whose area is 542.7 cm.² and one dimension 215 cm.
8. Construct a parallelogram. Use either side and find the corresponding altitude. From these find its area. Repeat for the other side. Compare the areas found.
9. What will be the cost at 28 ¢ per square yard to plaster a room 12 ft. by 16 ft. which is 8.5 ft. high?
10. What is the price per square foot of a rug 8' 3" by 10' 6" sold at \$45.75?
11. What is the price per square foot of a rug 9' by 12' sold for \$49.75? If the quality of the two rugs is the same, which rug is the more economical?
12. Construct two equal parallelograms and cut each from paper. Divide one along a diagonal. Compare the areas of the two triangles thus formed. Divide the other parallelogram along the other diagonal. Again compare the areas. What do you find? What is your conclusion?

155. Areas of Triangles.—Compare the area of the triangle ABC and the area of the parallelogram $ABCD$.

The altitude of a triangle is defined as the perpendicular distance from any vertex to the opposite side which is called the base.



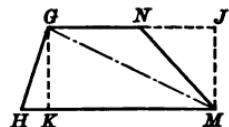
From the figure it is seen that the altitudes of the triangle ABC and of the parallelogram $ABCD$ are both h . The area of the triangle is $\frac{1}{2}$ that of the parallelogram, hence,

$$A = \frac{1}{2}bh. \quad (1)$$

156. Areas of Trapezoids.—A trapezoid, as $MNGH$, can be divided into triangles along either diagonal MG , or NH . Suppose it is divided along MG .

Then the altitude of the triangle MHG is GK , and the altitude of the triangle MGN is MJ . These altitudes are of the same length. Why? The area of the trapezoid is the sum of the areas of the two triangles. Since the triangles have equal altitudes, the sum of their areas will be

$$A = \frac{1}{2}(b + B)h, \quad (2)$$



b and B being the lengths of the parallel sides. Point out the lengths b , B , and h in the figure.

EXERCISES

1. Construct a triangle with sides 3, 4, and 5 inches. Use each side as a base and measure the corresponding altitude. Compute the area using each base and the corresponding altitude. What is the conclusion?

2. Construct a triangle with sides of any length. Repeat the measurements of Ex. 1. How do you account for any slight variations in your results? What is the conclusion?

3. A triangular pennant 24" long by 9" wide contains what fraction of a yard? How many can be cut from 2 yd. of felt 48" wide?

4. Express the altitude of a triangle in terms of its area and base from equation (1), page 140; express the base in terms of its area and altitude.

5. Find to two decimal places, the altitude of a triangle whose area is 2,175 sq. in. and base 314 in.

6. Find to two decimal places, the base of a triangle whose area is 315 sq. dm. and altitude 107 dm.

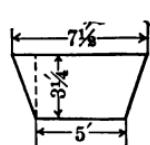
7. A summer cottage is 18' by 24' and 8' 4" high. The gable is on the narrow part and the ridge is 3' 4" above the eaves. Find the area of the surface to be painted, making no allowance for windows.



8. One gallon of paint is supposed to cover 250 sq. ft. How many gallons will be required to paint the above cottage?

9. How many rectangles 12" by 15" can be cut from a piece of cloth a yard wide and 3 yd. long? Waste as little as possible.

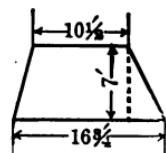
10. The parallel sides of a trapezoid are 15.7' and 23.8'. Find to two decimal places the area, if its altitude is 18.3'.



11. What is the area of the cross-section of a drainage ditch as here shown?

12. A railroad embankment was constructed in the form of a trapezoid. What is the area of its cross-section having the dimensions as shown.

13. Play a number game, using decimals



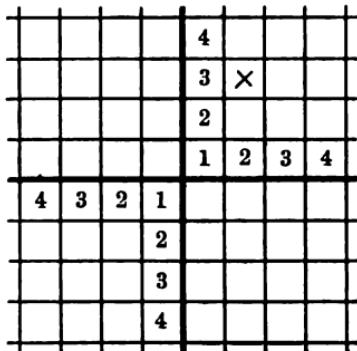


Fig. I

X	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Fig. II

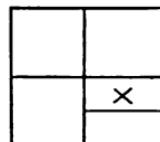


Fig. III

157. Land Measure.—The first rectangular public land survey was made in Ohio in 1785. During the succeeding twenty years some changes were made. Since that time the public land surveys west of New York and Pennsylvania have been made in the manner described below.

At any convenient place the surveyors first laid out a true north and south line called the **principal meridian**. Through this point a true east and west line was next laid out called **principal base line**.

Every six miles **township lines** were constructed parallel to the base line. Every six miles **range lines** were constructed parallel to the principal meridian. These lines, as shown above, divide the land into squares of six miles each way, called townships; see Fig. I.

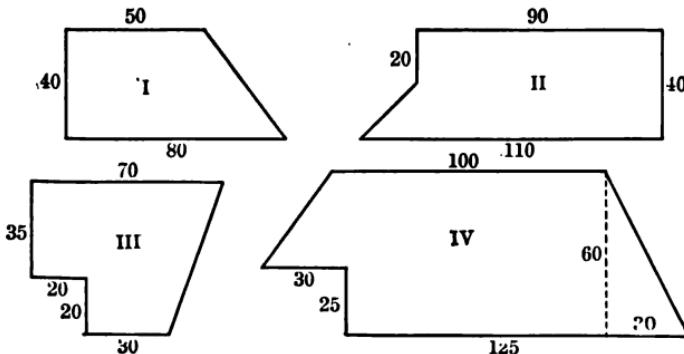
Each township is divided by lines a mile apart into 36 sections of 640 acres each. The sections of a township are numbered as shown in Fig. II. Sections are divided into quarter-sections which are again divided into halves and quarters; see Fig. III.

A piece of land is located—as in a deed—by stating the part of a section, the section number, township, and range. Thus, for the above:

N. $\frac{1}{2}$ S.E. $\frac{1}{4}$ Sec. 6, T. 3 N., R. 2 E.

EXERCISES

1. When was the present system of measuring land in the United States established? How long ago was that?
 2. A farmer owns the north one-half of section 17 in township 2 north and range 4 west. Locate this on the map and describe it as on the last page. What is its value at \$85 per acre?
 3. Locate and find the price at \$67.50 per acre:
S. $\frac{1}{2}$ N.W. $\frac{1}{4}$ Sec. 8, T. 5 S., R. 4 W.
 4. Locate and find the price at \$97.50 per acre:
N.E. $\frac{1}{4}$ S.E. $\frac{1}{4}$ Sec. 11, T. 2 N., R. 5 W.
 5. A farmer owning a section sold the N. $\frac{1}{2}$ of N.W. $\frac{1}{4}$ for \$75 per acre. What did he receive? How much land had he left? Draw a section and locate the part sold.
- 158. Irregular Areas.**—In the eastern part of the United States land is laid out in shapes as shown below. If the lengths are given in rods, find the area in acres by dividing each piece into rectangles, triangles, or trapezoids.



159. Circumferences of Circles.—For hundreds of years men tried to “square the circle.” That is, they tried to find a square whose area would equal that of a circle. This has been proved impossible.

A circle rolled completely around once, will pass over a length nearly $\frac{22}{7}$ times its diameter. This constant number, $\frac{22}{7}$, by which the diameter must be multiplied to get the length of the circumference cannot be found exactly but it can be approximated to any number of decimal places. Ludolph van Ceulen computed it to thirty-five decimal places and William Shank to seven hundred seven decimal places. Its first fifteen decimal places are 3.141592653589793. Ordinarily use $\frac{22}{7}$; when four decimal places are needed in the result use 3.1416. The Greek letter π (*pi*) is the usual symbol for this constant. Then,

$$C = 2\pi R. \quad (1)$$

EXERCISES

1. Carry out the above suggestion for finding the value of π . How much does your value vary from $\frac{22}{7}$? What is your per cent of error?
2. Using π as $\frac{22}{7}$ find the circumference of the circle having a radius of 4 in.; of 5 dm.; of 6.2 ft.; of 9 m.
3. Using π as 3.1416 find the circumferences of the circles in Ex. 2.
4. Play a number game, using weights and measures.



160. Areas of Circles.—Suppose that a circle is divided into semicircles and these divided by cuts along radii, as in Fig. I. The two semicircles can be placed together so as to form a parallelogram as in Fig. II. The base of this parallelogram is a semicircumference, πR , and its altitude is R . What is the area of a parallelogram? Hence, the area of the circle is

$$A = \pi R \times R \quad (1)$$

$$= \pi R^2. \quad (2)$$

The equations on this and the preceding page are proved much more exactly in higher mathematics.



Fig. I

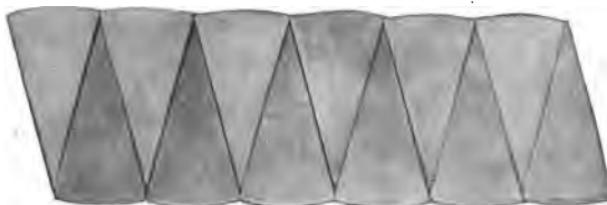


Fig. II

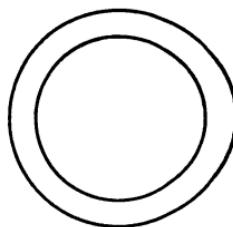
EXERCISES

1. Using π as $\frac{22}{7}$ find the area of the circle having a radius of 5 in.; of 7 cm.; of 2.4 ft.; of .25 m.
2. Using π as 3.1416 find the areas of the circles in Ex. 1.
3. Find R in terms of C and π from (1), page 144.
4. Find the radius to two decimal places of the circle whose circumference is 12 ft.; 18 dm.; 45 rd.; 16 m.
5. Find the radius of a circular quarter-mile track.
6. How many revolutions per mile will a 30 in. automobile wheel make?
7. Measure the circumference of a tree with a tape line. From this compute the diameter of the tree.

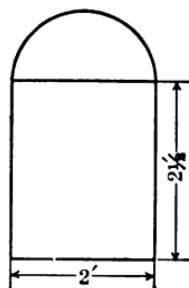
8. A teakettle has a circular bottom with diameter 8.3 in. How many square inches of copper are in its bottom?

9. The radius of a circular city park is 180 ft. What is the area of a 5 ft. sidewalk around it?

10. Two boys ran a race about this park. How much farther did the boy run who was on the outer side of the walk than the boy who was on the inner side of the walk?



11. Find the area of the glass in the window here shown.



12. In a properly constructed hot-air furnace the area of the cold air return pipe should be $\frac{5}{6}$ the total areas of the hot-air pipes. What must be the area of the cold-air return pipe for a house with 5 hot-air pipes having a radius of 6 in., and 2 hot-air pipes having a radius of 8 in.?

161. **Involution.**—Raising a number to a power is generally spoken of as **involution**. What is the law of exponents in multiplication?

EXERCISES

Carry out as many of the following operations without pencil and paper as possible:

- | | | |
|---------------------|---------------------------------|---|
| 1. 3^3 | 7. 50^2 | 13. $5^2 \times 2m^3 \times m^4$ |
| 2. 2^3 | 8. 125^2 | 14. $(3a^2)^3$ or
$3a^2 \times 3a^2 \times 3a^2$ |
| 3. 3^4 | 9. $3a^2 \times a^3$ | 15. $2^4 \times 5a^3 \times 5a^3$ |
| 4. 9^2 | 10. $5g^2 \times 4g$ | 16. $5^2 \times 2^4 \times 7^3$ |
| 5. 4×5^3 | 11. $2^3 \times g^5 \times g^2$ | 17. $(\frac{1}{2}k)^2$ |
| 6. $2^2 \times 3^2$ | 12. $4w^7 \times 3w^5$ | |

18. Let w be width and h height of the above window. Express its area by a formula.

162. Evolution.—Finding the three equal factors of 27, the five equal factors of 32, or the four equal factors of 625, is just the opposite of finding 3^3 , 2^5 , or 5^4 . This process is called **evolution**.

Corresponding to the square, the cube, the fourth power, and so on, there is the **square root**, the **cube root**, the **fourth root**, and so on.

By the law of exponents,

$$a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} = a, \quad (1) \quad \text{See Art. 105.}$$

and $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8, \quad (2)$

but $2 \times 2 \times 2 = 8. \quad (3)$

Hence, $8^{\frac{1}{3}} = 2. \quad (4)$

Likewise $25^{\frac{1}{3}} = 5$; $36^{\frac{1}{3}} = 6$; $81^{\frac{1}{3}} = 3$; $32^{\frac{1}{3}} = 2$.

Explain each of these statements.

Using these terms, $27^{\frac{1}{3}}$ is the cube root of 27; $16^{\frac{1}{2}}$ is the square root of 16, and so on.

EXERCISES

Give the following results as quickly as possible:

- | | | | |
|-----------------------|------------------------|-----------------------|-------------------------|
| 1. $9^{\frac{1}{3}}$ | 4. $125^{\frac{1}{3}}$ | 7. $81^{\frac{1}{3}}$ | 10. $216^{\frac{1}{3}}$ |
| 2. $49^{\frac{1}{2}}$ | 5. $64^{\frac{1}{3}}$ | 8. $64^{\frac{1}{3}}$ | 11. $64^{\frac{1}{2}}$ |
| 3. $27^{\frac{1}{3}}$ | 6. $16^{\frac{1}{2}}$ | 9. $16^{\frac{1}{3}}$ | 12. $81^{\frac{1}{3}}$ |

13. Solve equation (2) on page 145 for R^2 .
14. The area of a circular disk is 154 sq. in. Find the square of its radius, and from this its radius.
15. It required $28\frac{2}{7}$ sq. in. to make a circular copper disk. Find the radius of the disk.
16. Express the area of a square in terms of its length.
17. Express the length of a square in terms of its area.
18. Express the area of a circle in terms of its diameter.
19. Express the diameter of a circle in terms of its area.
20. Express the radius of a circle in terms of its area.

163. Radical Signs.—Square roots are often indicated thus, $\sqrt{25}$ or $\sqrt{36}$. Higher roots are indicated by writing the root index in the $\sqrt[•]{}$ part of the radical sign. Thus:

$$\sqrt[3]{8}; \sqrt[3]{125}; \sqrt[5]{32}; \text{ etc.}$$

The simpler fractional exponent is much to be preferred and will be used mostly in the future.

EXERCISES

Change the following to exponential forms and find the required roots:

1. $\sqrt{4}$	4. $\sqrt{16}$	7. $\sqrt[4]{32}$	10. $\sqrt[5]{125}$
2. $\sqrt[3]{8}$	5. $\sqrt[4]{16}$	8. $\sqrt[4]{625}$	11. $\sqrt[5]{81}$
3. $\sqrt{625}$	6. $\sqrt[3]{27}$	9. $\sqrt[4]{64}$	12. $\sqrt[5]{243}$

Write the following by use of radical signs and find the required roots:

13. $49^{\frac{1}{2}}$	16. $64^{\frac{1}{3}}$	19. $81^{\frac{1}{4}}$	22. $125^{\frac{1}{5}}$
14. $16^{\frac{1}{4}}$	17. $27^{\frac{1}{3}}$	20. $8^{\frac{1}{2}}$	23. $9^{\frac{1}{2}}$
15. $25^{\frac{1}{2}}$	18. $81^{\frac{1}{4}}$	21. $64^{\frac{1}{5}}$	24. $625^{\frac{1}{4}}$

25. Find the side of the square whose area is 25 sq. ft.; 49 cm.²; 81 dm.²; 64 sq. in.

26. Find the side of the square whose area equals the area of a rectangle 12 in. by 3 in.; 16 dm. by 4 dm.

27. The following is an equation used by engineers:

$$D = \sqrt{\frac{H.P.}{6}}.$$

Find the value of D when $H.P. = 150$; $H.P. = 294$.

28. From equation (2) on page 145 express R as a radical A and π under the radical sign.

164. Fractional Exponents with Numerators.—It has been shown that $8^{\frac{1}{2}}$ means 2. Hence,

$$8^{\frac{1}{2}} \times 8^{\frac{1}{2}} = 8^{\frac{1}{2}} \quad (1)$$

$$\text{and} \quad 2 \times 2 = 4. \quad (2)$$

$$\text{Hence, } 8^{\frac{1}{2}} \times 8^{\frac{1}{2}} = 8^{\frac{1}{2}} = 2 \times 2 = 4. \quad (3)$$

$$\text{Similarly, } 32^{\frac{1}{2}} = 16; 25^{\frac{1}{2}} = 3125; 27^{\frac{1}{2}} = 9, \text{ etc.} \quad (4)$$

Explain each of these fully.

A fractional exponent means that the root indicated by the denominator is to be taken and that number raised to the power indicated by the numerator.

EXERCISES

Find the following values:

$$1. \quad 125^{\frac{1}{3}} \quad 4. \quad 9^{\frac{1}{2}} \quad 7. \quad 16^{\frac{1}{4}} \quad 10. \quad 121^{\frac{1}{2}}$$

$$2. \quad 81^{\frac{1}{4}} \quad 5. \quad 27^{\frac{1}{3}} \quad 8. \quad 25^{\frac{1}{2}} \quad 11. \quad 16^{\frac{1}{3}}$$

$$3. \quad 4^{\frac{1}{2}} \quad 6. \quad 8^{\frac{1}{3}} \quad 9. \quad 8^{\frac{1}{4}} \quad 12. \quad 625^{\frac{1}{4}}$$

13. The equation stating the time t , in seconds, it takes any body to fall a distance d , in feet, is

$$t = \sqrt{\frac{d}{16}}.$$

Find t when d is 1600 ft.; d is 64 ft.; d is 400 ft.

14. Find the radius of the circle whose area is $\frac{550}{7}$ m.²; whose area is $\frac{1408}{7}$ sq. ft. See (2), page 145.

15. Play a number game, on raising to powers and extracting roots.

16. Turn to page 116 and express the length of AB in terms of BC and BD .

17. Find the length of AB if BC is 5 ft. and BD 20 ft.

18. Find the length of AB if BC is 4 cm. and BD 9 cm.

19. From equation (2), page 145, show that the area of a circle can be expressed by

$$A = .7854D^2.$$

20. Find the area of the circle whose diameter is 7 cm.; 4 in.; 12 ft.
21. From the equation in Ex. 19 express the diameter of a circle in terms of its area.
22. Find the diameter of a circle whose area is 314.16 sq. ft.; 28.2744 cu. dm.
23. The electrical engineer uses the formula

$$E = \sqrt{PR}.$$

Find the value of E when P is 4 and R is 9; when P is 25 and R is 4.

165. Square Roots of Numbers.—It is often necessary to find the square root of a number such as 5,634 or 87,256. Because only a few numbers are the squares of others, the exact square root of a number usually cannot be found. An approximate square root can, however, be found to as many decimal places as is desired.

Squares of numbers will give either twice as many digits as the number squared, or one less than twice as many digits. Thus, the square of 435 is 189,225, which has twice as many digits as 435. How many digits are there in the square of 142? of 812? of 216?

In order to decide how many digits there will be in the whole number of a square root, begin at the decimal point and divide the number into places of two digits in each. Thus, 87256 becomes 8'72'56. This shows that the root will have three digits. Similarly point off 45075 and 3240671. How many digits will the root of each have?

166. Method of Finding Square Roots.—Follow the form on page 151 closely in order to save yourself trouble. The principle underlying this form is too difficult for us to learn at this time.

First divide the number into places, as explained. Next find the largest digit whose square is just equal to or less than the first part; 8, of the number given. As 2 is the largest digit whose square is not larger than 8, it is placed above the 8 as the first digit of the root. The 2 is squared and the 4, thus obtained, subtracted from 8, leaving a remainder of 4. After this remainder, place the next two digits giving 472. As one more number is now used there are in reality two number places in the root. It is not 2 but 20 which has been found. This 20 is doubled and the 40 put down as in the form. As 40 goes into 472 over 9 times, 9 is the next digit in the root. The 9 is added to 40 giving 49. This is multiplied by 9 and the product, 441, is subtracted from 472, just as 4 was subtracted from 8. Bringing down the next two digits gives 3,156 as the next dividend. As before the root is now not 29 but 290. This is again doubled and written down as the new divisor, 580. As 580 is contained over 5 times in 3,156 the next digit in the root is 5. The process is now repeated.

By annexing places of two zeros, the root of a number may be found to any number of decimal places. In the above, bringing down the 00 gives 23,100. The new divisor is twice 2,950, or 5,900, which goes over 3 times into 23,100. The 3, with a decimal point before it, is the next digit in the root. The process is now repeated for as many decimal places as are needed.

$$\begin{array}{r}
 295.3 \\
 \hline
 872'56.00 \\
 4 \\
 \hline
 40 \quad | \quad 472 \\
 9 \\
 \hline
 49 \quad | \quad 441 \\
 580 \quad | \quad 3156 \\
 5 \\
 \hline
 585 \quad | \quad 2925 \\
 5900 \quad | \quad 23100 \\
 3 \\
 \hline
 5903 \quad | \quad 17709 \\
 \hline
 & & 5391
 \end{array}$$

EXERCISES

Find the square root of the following to two decimal places. Check your work by squaring the number found and compare with the number given:

- | | | |
|---------------|---------------|-------------|
| 1. 23.3456 | 5. 3,508.3057 | 9. 5,604 |
| 2. 624.3416 | 6. 1,760.1604 | 10. 234,512 |
| 3. 809.1604 | 7. 83,506 | 11. 134,504 |
| 4. 4,567.2318 | 8. 17,084 | 12. 40,671 |

13. Find the radius of the cold-air pipe in Ex. 12, page 146.

167. Right Triangle.—Nearly two thousand years ago the great Greek mathematician Pythagoras discovered the following truth: The area of the square drawn upon the hypotenuse of a right triangle equals the sum of the areas of the squares drawn upon the two perpendicular sides. In symbols this is:

$$c^2 = a^2 + b^2, \quad (1)$$

$$\text{hence, } c = (a^2 + b^2)^{\frac{1}{2}}. \quad (2)$$

If a^2 is subtracted from both sides of the equation (1),

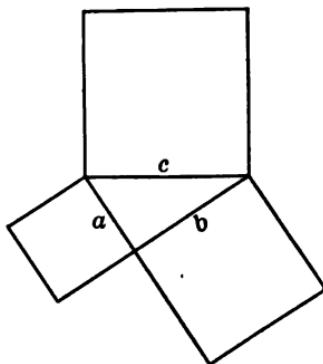
$$c^2 - a^2 = b^2 \quad (3)$$

$$\text{and } (c^2 - a^2)^{\frac{1}{2}} = b. \quad (4)$$

Hence, if any two sides of a right triangle are known, the remaining side can be found.

EXERCISES

- Find the hypotenuse of the right triangle whose perpendicular sides are 4 cm. and 3 cm.; 5 ft. and 12 ft.
- The hypotenuse of a right triangle is 37 in. and one of the perpendicular sides 12 in. Find the third side.
- The baseball "diamond" is a 90 ft. square. Find the length of its diagonals.
- In a baseball game Henry caught a ball on the right foul line 120 ft. back of first base. How far must he throw the ball to the home plate? to second base? to third base? Show by a drawing.



5. In order to keep a screen door from sagging, a wire is stretched from the inner upper corner to the lower outer corner. What length of wire is needed for a door 7' 3" by 2' 8"? for a door 8' by 3' 2"?

6. What is the diagonal distance across a section of land?

7. A steel tower at a manufacturing plant is held in position by four guy-cables each attached to the tower 50 ft. above the ground. The other end of each cable is attached to a post driven into the ground. The post for the first cable is 90 ft. from a point directly under the point to which it is attached to the tower; the second 100 ft. away; the third 120 ft. away; and the fourth 140 ft. away. Find the length of each cable.

8. If R_3 is the radius of the circle whose area equals the sum of the areas of the circles whose radii are R_1 and R_2 , then

$$R_3^2 = R_1^2 + R_2^2.$$

Hence,

$$R_3 = (R_1^2 + R_2^2)^{\frac{1}{2}}.$$

Find the radius of the circle whose area equals that of two circles with radii 6 in. and 2.5 in.

9. Estimate the length of a diagonal on the floor of your schoolroom. Measure the length and width, and compute the length of the diagonal. What was the error of your estimate? What was the per cent of error?

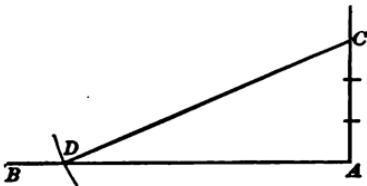
10. Elsie has a piece of cloth that she wishes to cut into bias strips for trimmings. What will be the length of the strip facing on the diagonal of a piece of cloth which is a square 30 in. on each side?

11. Express by an equation that in a square the diagonal squared is twice the square of a side.

12. Play a number game, using square roots.

168. Construction of Right Triangles.—All constructions of right triangles but one were treated on pages 126 and 127. Reread these pages if necessary.

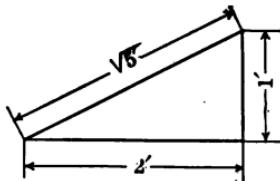
Suppose it is required to construct a right triangle with one perpendicular side 3 units and the hypotenuse 7 units long. Draw a line AB of any length, and at A erect a perpendicular AC , 3 units long. Strike an arc about C with the compass having a radius 7 units and cutting AB at D . Join C to D by a straight line. The required triangle is ADC . Why?



EXERCISES

1. Construct a right triangle with the perpendicular sides 2 in. and 1.25 in. Compute and measure the hypotenuse.
2. Construct a right triangle with one perpendicular side 1.2 in. and the hypotenuse 2.5 in. Compute and measure the remaining perpendicular side.
3. Construct a right triangle with one angle 35° and the side between this angle and the right angle 1.75 in.
4. The hypotenuse and one of the perpendicular sides of a right triangle are 8 cm. and 5 cm. Construct the triangle.
5. What is the length of the hypotenuse of the right triangle whose perpendicular sides are 5 cm. and 7 cm.?
6. Construct the above right triangle and measure its hypotenuse. What do you find? If the computation in Ex. 5 is considered correct, what is your per cent of error in construction?

169. Square Root Lengths.—Draftsmen and architects often must draw lines that are $5\frac{1}{2}$, $3\frac{1}{2}$, etc., units long. They do this by the construction of right triangles. If the perpendicular sides of a right triangle are 1 and 2 units long, the hypotenuse will be $5\frac{1}{2}$. Why? If the hypotenuse is 2 and one of the perpendicular sides is 1 unit long, the other perpendicular side will be $3\frac{1}{2}$. Why?



EXERCISES

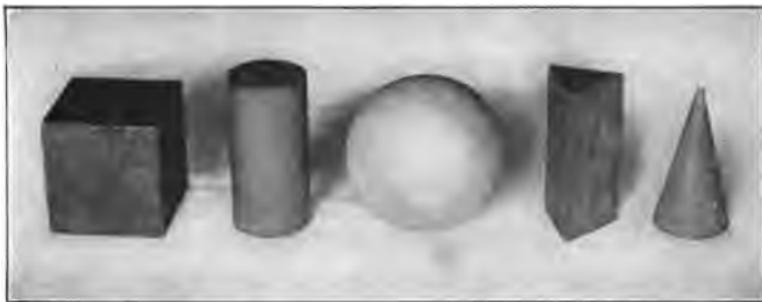
1. Draw a line that is $\sqrt{5}$ in. long.
2. Draw a line that is $\sqrt{2}$ cm. long.
3. Draw a line that is $\sqrt{7}$ cm. long.
4. Draw a line that is $\sqrt{10}$ cm. long.
5. What is the length of the diagonal walk through a square park 200 ft. on each side? How long a distance is saved by going through the park in the place of "around" it? What per cent of the distance around the park is saved by going through it?
6. Make the same computations as in Ex. 5, using any other length of side for the park. What do you find is the same in the two results? What does this prove?
7. Draw an equilateral triangle with sides 2 in. Draw the altitude. Compute the length of the altitude. What does the area of a triangle equal? Find this for the triangle you have drawn.
8. Find the area of the isosceles triangle whose equal sides are 13 m. and the third side 10 m.

9. A May pole is 9' high. How long ribbons are needed to be tied at the top of the pole in order that children 3' tall may start their frolic around the pole in a circle having a radius of 16'?
10. A garden is in the form of a square 4 rd. on each side. What length is each side in feet? Find the length of the sides of a square garden having double the area of this one.
11. Compute the length of the diagonal across the printed part of this page. Measure the distance with your ruler and compare the two results.
12. Surveyors use the truth of Pythagoras in laying out lines on the ground perpendicular to given lines. Suppose it is necessary to lay off a line at A , perpendicular to AC .



AB is first measured off 40 links by the surveyor's chain. One end of the chain is next fastened at A and the end of the 80th link at B . The chain is then pulled from the end of the 30th link until it is taut and this point marked D . In the triangle ADB the sides are $BA = 40$, $AD = 30$, and $BD = 50$. Hence $AD \perp AC$. Show that this is true.

13. If you do not have a surveyor's chain, use a heavy cord to lay out a line perpendicular to some line on the schoolground. In place of the link for lengths use the foot.



VIII

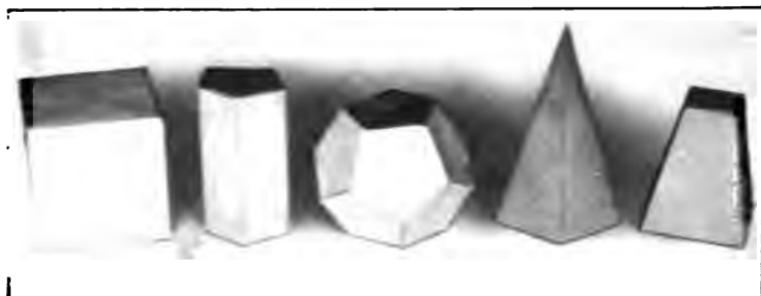
SOLID GEOMETRICAL FIGURES

170. Material Solids.—A ball, a piece of chalk, a box, or a house are all illustrations of material solids.

171. Mathematical Solids: Volumes.—Think of the space occupied by a material solid with the solid taken away. This is called a **geometrical solid**. The space occupied by the geometrical solid is called its **volume**.

EXERCISES

1. Which of the following are material solids: an ink-bottle; the ink in a bottle; the space occupied by an ink-bottle; a book; a leaf in a book; a page of a book; the surface of the floor; the air in the room ?
2. Give three illustrations of other material solids.
3. What is meant by the geometrical solid for each of the material solids of Ex. 1 ?
4. Give three illustrations of other geometrical solids.



172. Solids Bounded by Planes.—Planes bounding a solid, as the sides of a chalk-box, are called **faces** and their intersections are called **edges**. Each corner of the solid is called a **vertex**. Two or more are **vertices**. A line joining any vertex to another not in the same face, is called a **diagonal**.

EXERCISES

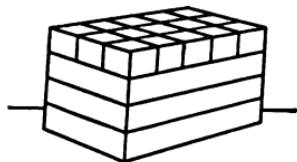
1. Which of the following are solids bounded by planes: a brick; a house; a ball; a cracker-box; a waste-basket; a book; the teacher's desk; a piece of chalk?
2. Give illustrations of three other solids bounded by planes.
3. A chalk-box has how many faces? how many edges? how many vertices? how many diagonals? How many edges meet at each vertex?

173. Surfaces of Rectangular Solids.—A solid bounded by planes which are all rectangles, as those of the ordinary box, is called a **rectangular solid**. The area of the total surface of a solid is the sum of the areas of each of its faces. The area of the surface is usually spoken of as the **surface**. Show that the surface of a rectangular solid is

$$S = 2 [wl + lh + hu].$$

Find the surface of a chalk-box.

174. Volumes of Rectangular Solids.—If the units shown in the figure are inches, there will be as many cubic inches in the top layer as there are square inches of surface in the top of the figure. The number of cubic inches in the top layer times the number of layers will give the total number of cubic inches in the volume. This is



$$V = lwh, \quad (1)$$

or

$$V = Bh, \quad (2)$$

in which B is the area of the base. If l , w , and h are given in feet, in what unit would the volume be given?

EXERCISES

1. Measure the length of each edge of a chalk-box.
2. Find the total surface of the box.
3. Find the volume of the box.
4. The dimensions of a box are 7.45 in., 8.5 in., and 6.25 in. Find the total surface to two decimal places using abbreviated multiplications.
5. Find the volume of the box in Ex. 4 to two decimal places using abbreviated multiplications.
6. A cubic foot contains about $7\frac{1}{2}$ gal. How many gallons will a tank contain that is 18 ft. long, $1\frac{1}{4}$ ft. wide, and $1\frac{3}{4}$ ft. deep?
7. It is estimated that a bushel of ear corn takes up $2\frac{1}{2}$ cu. ft. of space. How many bushels will a crib hold which is 8' by 12' by 40'?
8. Play a number game, using multiplication and division of decimals, and of common fractions.

8. A bushel of potatoes occupies about $1\frac{1}{2}$ cu. ft. How many bushels of potatoes can a commission merchant put into a bin 6' by 10' and 4' deep? To what depth must he pile the potatoes in this bin so as to have it contain 97 bu.?
9. What is the value of the potatoes in a bin 8' by 12' and 6' 8" deep at \$1.45 per bushel?
10. The altitude of a rectangular solid is 12.35 dm. and its volume is 45.75 dm³. Find the area of its base.
11. Solve equation (2), page 159 for h ; for B .
12. The base of a rectangular box is 45 cm². Find its height if the volume is 134 cm³; 563.72 cm³; 463.48 cc.
13. If $V = 3456$ cu. in. and $h = 215$ in., find B to two decimal places.
14. A cord of wood is 4' by 4' by 8'. How many cords are there in a pile of wood 16' long and 6' high in which the sticks are 4' long?
15. A schoolroom is supposed to have 240 cu. ft. of air for each occupant. Does a schoolroom 20' by 24' and 12' high seating 30 pupils fulfil this requirement?
16. Measure the length, width, and height of your schoolroom and compute its volume. Does it fulfil the requirement for Ex. 15?
17. Construct an open rectangular box with a base $1\frac{1}{2}$ in. by 2 in. and 3 in. high.
- For construction work, use a heavy paper which will fold over easily without breaking. Make very carefully all figures asked for and save them for future use.
18. Construct an open rectangular box which will have a volume of 8 cu. in. that will require the least possible material.

175. Diagonals of Rectangular Solids.—The figure shows that the diagonal AD is the hypotenuse of the right triangle ACD . Hence,

$$AD^2 = AC^2 + CD^2. \quad (1)$$

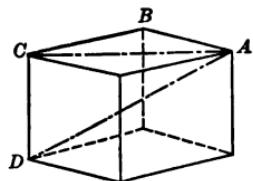
Looking at the top base, it is seen that AC is the hypotenuse of the right triangle ABC , hence,

$$AC^2 = AB^2 + BC^2. \quad (2)$$

By substituting this value of AC^2 in (1) we get

$$AD^2 = AB^2 + BC^2 + CD^2, \quad (3)$$

so $AD = (AB^2 + BC^2 + CD^2)^{\frac{1}{2}}. \quad (4)$



EXERCISES

1. Find the length of the diagonal of the rectangular solid, the edges of which are 3 cm., 4 cm., and 12 cm.
2. Make the measurements and compute the length of a diagonal in your schoolroom.
3. Compute the length of the diagonals of the box constructed for Ex. 17, page 160, by using the lengths of its edges. To verify the computations use a thin stick or knitting-needle to measure the length of the diagonal.
4. In the same way find the diagonals for the box constructed for Ex. 18 on page 160.
5. Find the diagonal of a cube whose edges are 1 in.; 6 cm.; 4.5 in.
6. Note that the diagonal of a cube is $3\frac{1}{2}$ times the edge of the cube. Express this as an equation.
7. Express by an equation the total surface, S , of a cube in terms of an edge, E ; the volume, V , in terms of an edge, E .

176. Lumber Measurements.—Lumber is measured by "board feet," usually called feet. A board foot has a surface of one square foot and a thickness of one inch or less. A board 10' long, 12" wide, and 1" or less thick contains 10 board feet; a board 10' long, 6" wide, and 1" or less thick contains 5 board feet. A piece of lumber 12' long, 12" wide, and 2" thick contains 24 board feet. These would be written: $1'' \times 12'' \times 10'$; $1'' \times 6'' \times 10'$; $2'' \times 12'' \times 12'$. Which dimension comes first? last? The number of board feet in a piece of lumber equals its length in feet times its width in feet times its thickness in inches. If l , w , and t are expressed in feet and fractions of a foot, then

$$Bf = 12lw t.$$

Show that the equation and the statement above correspond.

Flooring is usually 3", 4", or 6" wide. The groove uses up $\frac{3}{4}$ " so the width that really makes the floor is only $2\frac{1}{4}$ ", $3\frac{1}{4}$ ", and $5\frac{1}{4}$ " wide. Carpenters usually add $\frac{1}{4}$ of the floor space for waste.

All lumber comes in even foot lengths, as 8', 10', up to 22'. The longer lengths and the thicker lumber costs the more per board foot. These must be sawed from larger logs of which there are a less number. Clear fine-grained lumber is also more expensive than is coarse lumber containing knots.

177. Lumber Quotations.—The price of lumber is quoted on 1000 board feet. Thus, \$30 per M or \$45 per M . What is the meaning of M ? Sometimes lumber is also quoted on 100 board feet, as \$3 or \$4.50 per 100 feet.

Laths are $\frac{1}{4}'' \times 1\frac{1}{2}'' \times 4'$ and are sold by the bundle of 50 laths in each bundle.

The price of shingles is quoted per M or 4 bunches. See last of Art. 178.

EXERCISES

Find the number of board feet in each of the following nine pieces of lumber:

1. $1'' \times 12'' \times 12'$
 4. $2'' \times 8'' \times 16'$
 7. $4'' \times 6'' \times 14'$
 2. $1'' \times 6'' \times 16'$
 5. $4'' \times 4'' \times 12'$
 8. $6'' \times 8'' \times 18'$
 3. $2'' \times 6'' \times 14'$
 6. $2'' \times 6'' \times 14'$
 9. $\frac{3}{4}'' \times 6'' \times 16'$
10. How many board feet are there in 12 boards $1'' \times 12'' \times 12'$? What is the price at \$40 per *M*?
11. How many board feet are there in 15 scantlings $2'' \times 4'' \times 16'$? What is the lumber worth at \$37.50 per *M*?
12. How many board feet of $3''$ flooring will be needed to lay a floor $14' \times 16'$? Find the cost at \$60 per *M*.
13. How many board feet of $4''$ flooring will be needed to lay a floor $16' \times 18'$? Find the cost at \$65 per *M*.
14. What length of boards and how many are necessary to build a solid board fence 8' high around a lot 30' by 120'? How many board feet would this be and what would be the cost at \$32.50 per *M*?
15. Find the total for the following bill:

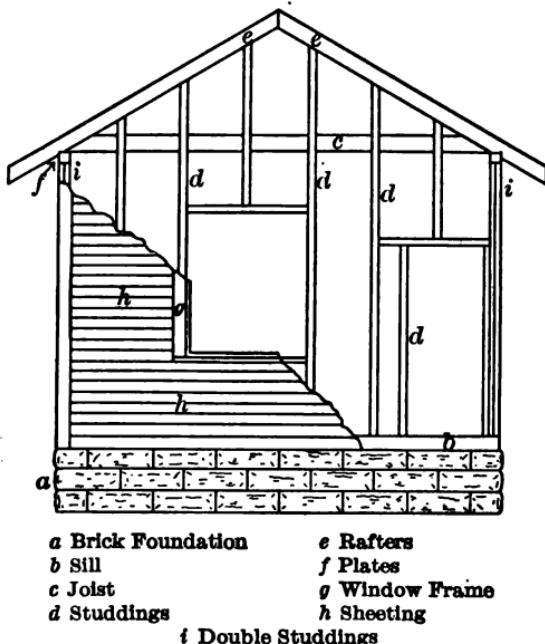
Thomas Brown Lumber Company

Osco, Ill., Jan. 29, 1919.

Terms: Due 30 days; 3% for cash. Sold to E. G. Clow.

48.....	$2'' \times 4'' \times 16'$	yellow pine No. 1.....	at \$ 37.50
64.....	$2'' \times 8'' \times 14'$	" " " 1.... "	42.50
240.....	$1'' \times 12'' \times 16'$	" " " 2.... "	35.00
1800 ft....	finishing	" " " 2.... "	60.00
600 ft....	$4''$ flooring—yellow pine	"	45.00

16. For how much can the bill be paid in cash?

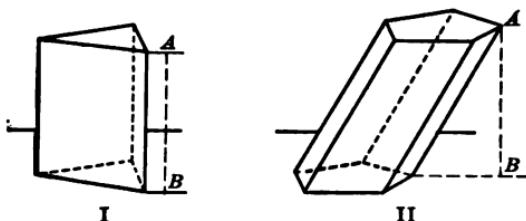


178. Lumber Terms.—Lumber used in different parts of the frame work of a house has the special names given in the picture. The sills are 4" x 6" or heavier. Joists are usually 2" x 6" and upright studdings 2" x 4". All lumber 2" x 4" or heavier comes under the general head of scantlings, although the name usually refers to 2" x 4". Boards a foot wide and over an inch thick are called planks. There are 2" and 3" planks, depending upon their thickness.

Shingles differ greatly in width. They are sold in bunches of 250, having an average width of 4". If the shingles in a bunch were laid in a row, side by side, how long would the row be? Carpenters figure 1000 shingles to the "square." A "square" is a square whose sides are 10' each. How many square feet are there in a "square"? Shingles laid 4 $\frac{1}{2}$ " to the weather have 4 $\frac{1}{2}$ " of the shingle uncovered.

EXERCISES

1. What would be the cost of 4" x 6" sills at \$ 50 per *M* for a cottage 16' by 32' and 10' high with the gables 4' above the eaves?
2. What would be the cost for the above cottage of 32 joists 2" x 6" x 16' at \$ 45 per *M*?
3. What would be the cost for the above cottage of the upright studdings placed 2' apart at \$ 45 per *M*? Take care to select the lengths for the ends of the house so as to give as little waste as possible. The cottage is divided into two rooms 16' x 16'.
4. Find the cost of 6" flooring for the cottage at \$ 55 per *M*.
5. Find the cost of the sheeting at \$ 32 per *M*. Make allowance for two doors 3' 6" by 6' 4" and five windows 2' 6" by 5' 4"; also to cover the roof.
6. What will be the cost at \$ 65 per *M* of siding the house with 6" siding laid $4\frac{1}{2}$ " to the weather?
7. The rafters, 2" x 4", are to project over the side walls 16". Compute the length of the rafters. Find the cost of the rafters placed 2' apart. Add to this the cost of six pieces for the ridge and the base of the house each 2" x 4".x 16'. The price is \$ 45 per *M*.
8. What will be the cost of the shingles at \$ 4.50 per *M*? Make use of the carpenter's estimate of the number of shingles needed for a "square."
9. Make a bill like the one on page 163, including the items mentioned in the above exercises. Use names of pupils in the class.
10. Play a number game, using addition and subtraction; casting out 9's.



179. Prisms.—A solid having two faces that are equal polygons in parallel planes, as the floor and the ceiling of a room, is called a **prism**. The two equal faces are its **bases** and may be triangles or polygons with any number of sides. All of the other faces will be parallelograms and are called **lateral faces**.

If the lateral edges are perpendicular to the bases, the prism is a **right prism**. See Fig. I above. The rectangular solids studied on pages 158–59 are the simplest right prisms. The cube is a prism whose faces are all squares.

180. Surfaces of Prisms.—The total surface of a prism is the sum of the areas of its bases and of its lateral faces. As an equation it is

$$S = A_b + A_l.$$

Tell the meaning of each symbol in the equation.

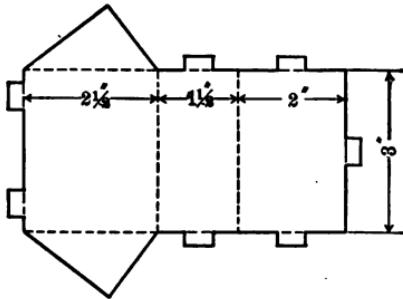
EXERCISES

1. Show that the figures at the top of the page are prisms.
2. Show that the room and a common box are right prisms. The bases of each are what kind of plane figures?
3. Give three illustrations of prisms.
4. What will be the cost to lath and plaster a room 12' by 16' and 9' high at 35 ¢ per square yard? Make no allowance for doors and windows.

181. Volume of Prisms.—The altitude or height of a prism is the perpendicular distance from any point in one base to the other base; as AB in the figures on the opposite page. If you have modeling clay in school, mold a prism having any parallelogram for bases and so as to lean like Fig. II on page 166. Write down its length, width, and altitude. Take the prism already molded and out of the clay in it, mold a rectangular solid like the one on page 159. Make the length and width of the rectangular solid the same as that of the prism. The two will have equal volumes. Why? They will have equal bases. Why? Compare their altitudes. Hence, equation (2), page 159, holds for prisms;

$$V = Bh.$$

EXERCISES

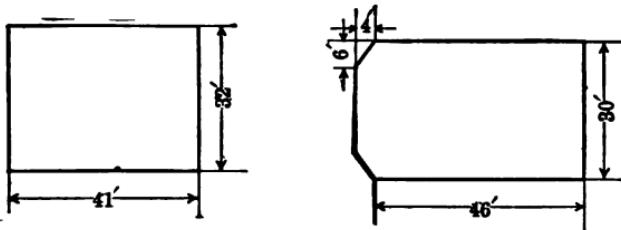


1. Draw above figure and cut from heavy paper. Then put together so as to form a prism. Leave one end open until you have solved Ex. 2.
2. Compute the volume of the prism constructed. Compute the volume of the box constructed for Ex. 17, page 160. Compare the volumes. Fill one with sawdust and pour into the other. Do the computations and measurements check?

3. The area of the base of a triangular prism is 8.5 sq. cm. Find its volume if the altitude is 9.4 cm.
4. The base of a prism is a right triangle whose perpendicular sides are 7 cm. and 6 cm. Find its volume if the altitude is 9 cm.

In hauling dirt, sand, and so on, a cubic yard is taken to be a load.

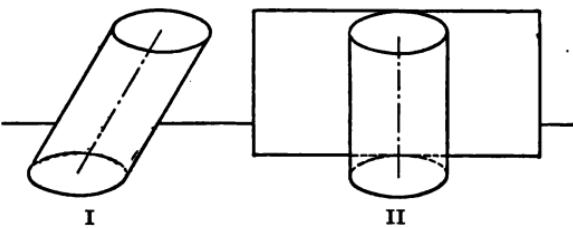
5. Find the number of loads of dirt that must be removed from the basements as shown, if excavations are to be $3\frac{1}{2}$ ft. deep. Find the cost of making the excavations at 69¢ per cubic yard.



6. From the equation on page 167 express the area of the base of a prism in terms of its volume and its altitude.
7. From the equation on page 167 express the altitude of a prism in terms of its volume and the area of its base.
8. The volume of a prism is 2018 cm^3 . If its base is a square each of whose sides is 13 cm., what is its altitude?
9. The volume of a prism is 180 cu. in. If the altitude is 5 in., what is the area of the base?
10. The base of a prism is a square. Find the length of the sides of the base if the volume is 823 cu. in. and the altitude 13 in.
11. The base of a prism is a triangle whose base and altitude are 8.4 cm. and 9.5 cm. If the volume is 356.7 cm^3 , what is the height of the prism?

182. Cylinders.—If the bases of a prism were replaced by circles and the lateral faces by a smooth curved surface, the solid would be a **cylinder**.

The line joining the centers of the two circular bases is the **axis** of the cylinder.



We shall study only cylinders in which the axis is perpendicular to the bases. Such are called **right circular cylinders**. The ordinary tin can is an illustration.

183. Surfaces of Cylinders.—A rectangle whose width equals the height of the cylinder and whose length equals the circumference of the cylinder will just envelope the cylinder. See Fig. II. The area of the curved surface is therefore equal to the height of the cylinder times the circumference of its base, or

$$A_c = 2\pi Rh. \quad (1)$$

The total surface is the curved surface plus that of the two bases and is then,

$$A_t = 2\pi Rh + 2\pi R^2, \quad (2)$$

or $A_t = 2\pi R (h + R). \quad (3)$

EXERCISES

1. Give three illustrations of cylinders.
2. Find the total area of a cylinder with altitude 6.5 in. and radius of its bases 2.3 in.
3. Find the total area of a cylinder with altitude 9.6 cm. and radius of its bases 4.5 cm.
4. Play a number game, using abbreviated multiplication of decimals.

184. Volumes of Cylinders.—A right cylinder may be thought of as a right prism having very very many sides. As the volume of the prism was found to be the area of the base times the altitude, the volume of the cylinder will also equal the area of its base times its altitude,

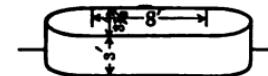
$$V = Bh, \quad (1)$$

or

$$V = \pi R^2 h. \quad (2)$$

EXERCISES

1. Find the volume of the cylinder whose altitude is 6.4 in. and the radius of its base 2 in.
2. Find the volume of a cylinder whose altitude is 5.4 in. and the radius of its base 2.7 in.
3. Find the volume of a cylinder with the radius of its base 7.5 cm. and its altitude 12 cm.
4. If a cubic foot contains about $7\frac{1}{2}$ gal., how many gallons will a milk can hold having a radius of its base 5 in. and an altitude of $2\frac{1}{2}$ ft.?
5. How high must a milk can be made to hold 10 gal. if the radius of its base is to be 8 in.?
6. Find the volume of the water tank shown in the figure. It is in the form of a rectangle with ends semi-circles. If a cubic foot holds about $7\frac{1}{2}$ gal., how many gallons will the tank hold?
7. Find the total areas of the cylinders in Exs. 2 and 3.
8. A baking-powder can has a radius of 2 in. and an altitude of 5 in. How large a label will be needed to cover the curved surface?
9. If the cylinders in Exs. 1, 2, and 3 above were tin cans, what size labels would be needed for each? Find the area of each label.



10. An open cylindrical tank has a radius of 3' and a depth of 6'. How many cubic feet of water will it hold? How many gallons is this?

11. How many gallons will a cylindrical cistern hold which has a radius of 2.5' and a depth of 7'? How many barrels of $31\frac{1}{2}$ gal. each will it hold?

12. Tell how to find the number of barrels a cylindrical cistern of a stated radius and depth will hold.

13. Express your statement of Ex. 12 in the form of an equation.

14. How deep should a cylindrical cistern be made to hold 60 barrels and have a radius of $2\frac{1}{2}'$?

15. Tell how to find the depth of a cylindrical cistern which will hold a certain number of barrels and have a given radius.

16. Express your statement of Ex. 15 in the form of an equation.

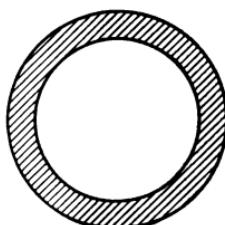
17. How deep should a cylindrical cistern be to hold 40 barrels, if its radius is 3'?

18. Find the radius of a cylindrical cistern which will hold 60 barrels and have a depth of 10'.

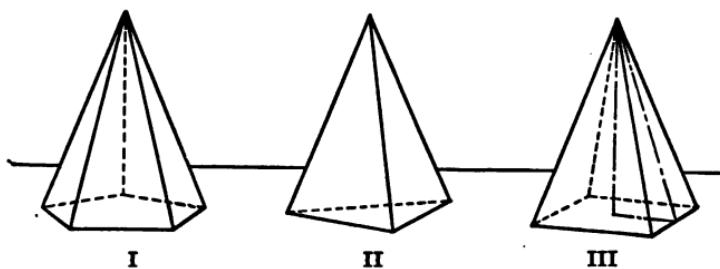
19. Tell how to find the radius of a cylindrical cistern which is to hold a certain number of barrels and which is to have a given depth. Express this as an equation.

20. The inner diameter of an iron pipe is 6" and the outer diameter is $6\frac{1}{2}"$. Find the number of cubic feet of iron in a pipe that is 60' long. See figure.

21. A rectangular piece of wood 2.5" x 2.5" x 10" is turned on a lathe into a rolling-pin. How much wood is cut off?



- 22.** The outer diameter of the pipes in a radiator is 1.5''. In a radiator there are 76 pipes each 2' 8'' long. Find the area of the entire radiating surface.
- 23.** Find the area of the inner surface of the tank in Ex. 10 and the cost of constructing it at 39 ¢ per square yard.
- 24.** Find the inner surface of the cistern in Ex. 11 and the cost of constructing it at 41 ¢ per square yard.
- 25.** A farmer had a cylindrical silo constructed 28' high with a radius of 12'. Find the amount of concrete surface required in its construction.
- 26.** If a cubic foot of corn silage weighs 38 lb., how many tons of silage would be required to fill the silo of Ex. 25 ?



185. Pyramids.—Pyramids have a polygon as a base and triangles for lateral faces. All these triangles meet in a point called the **vertex**. The perpendicular distance from the vertex to the plane of the base is the **altitude** or the height of the pyramid. The perpendicular distance from the vertex to the base of any triangular face is the **slant height** of that face. If the base is a regular polygon and the faces are all isosceles triangles, the pyramid is a **regular pyramid** and the slant height of each face triangle will be the same. The intersection of any two face triangles is called an **edge**. Only regular pyramids will be studied.

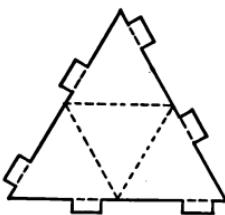


186. The above picture shows the famous pyramids of Egypt which were constructed over 5000 years ago and used as tombs. The largest was 479 ft. high and had a square base of 764 ft. It has since been decreased in size by removing stone for building purposes.

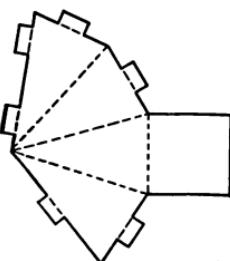
EXERCISES

- 1.** Point out the faces, edges, vertices, and slant height in Fig. III on page 172.

- 2.** Construct an equilateral triangle with sides 3 in. Construct three isosceles triangles as in the picture. Put these triangles together to form a triangular pyramid.



- 3.** Construct a square with sides 2.5 in. Construct four isosceles triangles as in the picture. Put these together to form a pyramid.



- 4.** Construct pyramids similar to those in Exs. 2 and 3 with other lengths of sides.

- 5.** Ascertain the height of some tall building in your vicinity. Compare this with the height of the Egyptian pyramid mentioned above.

187. Surfaces of Pyramids.—The area of the lateral surfaces of a pyramid equals the sum of the areas of the face-triangles. In regular pyramids the slant height is the same for each face-triangle. Therefore, if the number of faces is n , the length of the slant height s , and the length of the base edges b , the area of the lateral surface is

$$L_a = \frac{1}{2} nbs. \quad (1)$$

If the perimeter of the base is P , then

$$L_a = \frac{1}{2} Ps. \quad (2)$$

The total surface is the sum of the areas of the base and of the lateral faces, or

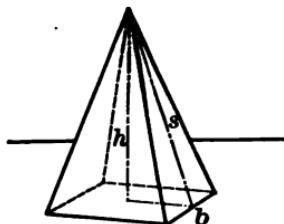
$$T_a = L_a + B_a. \quad (3)$$

$$T_a = \frac{1}{2} Ps + B_a. \quad (4)$$

EXERCISES

1. Explain equations (3) and (4).
2. A regular pyramid has a square base with sides 8.6 in. and slant height 9 in. Find its total area.
3. The base of a regular pyramid is a pentagon whose sides are 7 cm. each. Find the lateral area of the pyramid if the slant height is 13 cm.
4. Find the lateral area of a regular pyramid whose base is a hexagon with sides 3.5 in. and slant height 7 in.

Make a drawing to show the relation that exists between the base edge, the altitude, and the slant height of a right square pyramid. Find the slant height and lateral area of the Egyptian pyramid mentioned on page 173. Carry computations only to the nearest whole number.



6. The hopper at a mill is in the form of a pyramid with a square opening 4' x 4' and a depth of 5'. Find the amount of lumber needed for its construction.

7. Show by a drawing the relation between length of base edge, an edge, and slant height of a pyramid. Find the slant height of the pyramid whose edges are 15' and base edges each 18'.

8. A house has a tower in the form of a regular pyramid. Its base is a hexagon with sides each 5'. The slant height is 11.5'. Find the area of the roof of the tower and the cost of the shingles to cover it at \$ 5.00 per *M*. Use the carpenters' rule that *M* shingles cover a "square." Add 20 % for waste in cutting the shingles.

9. In a similar tower the base is a hexagon with sides each 10' and edges 13'. Find the area of the roof and the cost of the shingles at \$ 4.50 per *M*. Add 20 % for waste.

10. A pyramid has a square base with each side 24'. Its altitude is 13.5'. Find its lateral area.

11. Solve equation (1), page 174 for *s*; for *b*.

12. The lateral surface of a regular pyramid is 75 sq. in. If the base is a regular pentagon whose sides are each 6 in., what is its slant height?

13. A regular pyramid whose base is a square has a lateral surface of 70 sq. in. If its slant height is 7 in., what is the length of each side of the square base?

14. The lateral surface of a regular pyramid is 180.6 dm². The base is a hexagon each side being 4.3 dm. Find its slant height.

15. A tin cap is needed for a roof in the form of a regular pyramid of four sides. The altitude of the pyramid is to be 9' and each side of the base 12'. Find the amount of tin needed to construct the cap.

188. Volumes of Pyramids.—If you have modeling clay, construct a right prism. Place the prism constructed upon a sheet of paper and draw around the base. Then measure its altitude and write this down. Divide the clay into three equal parts. With one of the three equal parts construct a pyramid having a base equal to the base of the prism. This shows that the volume of a pyramid is $\frac{1}{3}$ the volume of a prism having the same base and an equal altitude. How? As the volume of the prism was

$$V = Bh, \quad (1)$$

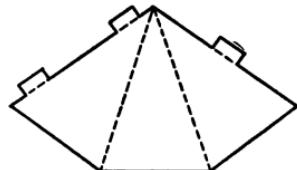
that of the pyramid is

$$V = \frac{1}{3} Bh. \quad (2)$$

Later in the study of geometry you will prove much more exactly the truth of equation (2).

EXERCISES

1. Draw on heavy paper and cut out the form here shown. Make the equal sides of the isosceles triangles about 4 in. and the bases of the triangles about 2 in. Fasten the outer edges together so as to form a pyramid open at the base.



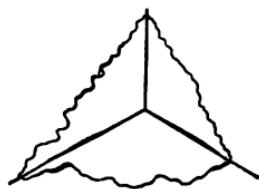
2. Construct an open prism having a base and an altitude equal to that of the pyramid constructed in Ex. 1.

3. Fill the pyramid constructed in Ex. 1 with sawdust and pour the same into the prism constructed for Ex. 2. Repeat until the prism is full. What does this prove?

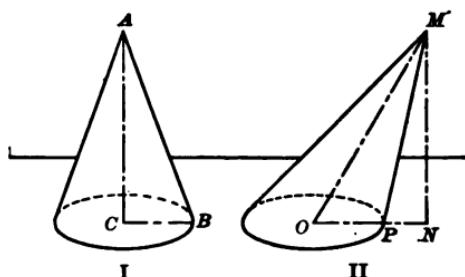
4. A pyramid has an altitude of 10 in. and the area of its base is 54 sq. in. Find its volume.

5. Find the volumes of the pyramids in Exs. 6, 10 and 15 on page 175.

- 6.** Mr. Jarus, who is moving away, sells the coal in his bin to his neighbor. The coal forms a pyramid in a corner of the bin. The three measures along the edges from the corner are $7' \times 8' \times 9'$. If 35 cu. ft. make a ton, how many tons does Mr. Jarus have to sell?



- 7.** Solve equation (2), page 176, for B ; for h .
- 8.** The area of the base of a pyramid is 32 sq. in. Find its altitude if the volume is 45 cu. in.
- 9.** The volume of a pyramid is 28 cu. in. Find the area of the base if the altitude is 12 in.
- 10.** The volume of a pyramid with a square base is 270 cm^3 . If the altitude is 7.5 cm., what is the area of the base? What is the length of each side of the base?
- 11.** Find the height of the pyramid whose volume is 23.5 cu. ft. and the area of its base 8.6 sq. ft.
- 12.** A pyramid has a slant height of 13 dm. and a square base whose sides are each 10 dm. Find its altitude and its volume.
- 13.** Find the volume of the pyramid having a square base with sides each 8 ft. and slant height 7 ft.
- 14.** A bushel of grain, as wheat or oats, occupies about $1\frac{1}{4}$ cu. ft. Oats in a bin occupies a corner like the coal in Ex. 6, for which the measurements are $8' \times 11' \times 15'$. Find the number of bushels this pyramid contains.
- 15.** Find the volume of the Egyptian pyramid mentioned on page 173.
- 16.** Find the contents by measurements as in Ex. 6 of a heap of coal, grain, or like substance placed in a corner.



189. Cones.—Cones may be thought of as coming from pyramids by making the base a circle and the lateral surface a curved surface. The **altitude** is the perpendicular distance from the vertex to the plane of the base. The line from the vertex to the centre of the base is the **axis**. A **right cone** is one in which the axis is perpendicular to the base; the axis is therefore the altitude of the right cone. Only right cones will be studied here.

190. Surfaces of Cones.—The lateral surface of a cone is found just as the surface of the regular pyramid was found. The perimeter of the base is now the circumference. Therefore,

$$L_a = \frac{1}{2} s \times 2\pi R \quad (1)$$

$$= \pi R s. \quad (2)$$

The total area of the surface equals the lateral area plus the area of the base, or

$$T_a = L_a + B \quad (3)$$

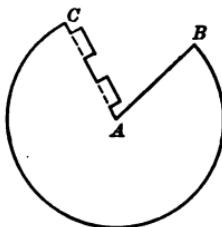
$$= \pi R s + \pi R^2 \quad (4)$$

$$= \pi R(s + R). \quad (5)$$

EXERCISES

1. Explain fully the above formulas.
2. The slant height of a cone is 8 dm. and the radius of its base 4 dm. Find its lateral area.

3. Draw a circle upon heavy paper and cut out a part as shown in the picture. Bring the lines AB and AC together and fasten them. Measure the radius, slant height, and altitude of the cone you have made. Find its lateral area.



4. Find the total area of the cone which has an altitude of 15" and the radius of its base 3.5".

5. A tepee tent with a diameter of 12' was found to have a slant height of 10'. How many yards of material 29" wide are there in the tent?

6. The radius of the base of a cone is 13" and its lateral area is 132 sq. in. Find its slant height.

7. What kind of a triangle is ABC , in Fig. I on page 178? If the altitude is 4 cm. and the radius of its base is 3 cm., what is the slant height?

8. How are the lengths of slant height, altitude, and radius of the base of a cone connected? Express this by an equation.

9. If the altitude of a cone is 7" and the radius of its base is 3", what is its slant height?

10. The radius of the base of a cone is 5" and its slant height 13", what is its altitude?

11. The radius of the base of a cone is 12 dm. and the altitude is 350 cm. Find its slant height and its lateral area.

12. A tower on a house has a diameter of 9' and an altitude of 6'. Find the total surface of the roof. What will be the cost of the shingles for it at \$ 5.50 per M ? Allow 35% for waste.

13. A cone has a slant height of 15" and an altitude of 8". Find the lateral area correct to two decimal places.

191. Volumes of Cones.—If there is modeling clay in the school, construct a circular cylinder with any base and altitude. Divide the cylinder into three equal parts and mold one part into a cone having the same circle for base as the cylinder. The cone will be found to have the same altitude as the cylinder. The volume of the cylinder being Bh , the volume of the cone will be

$$V = \frac{1}{3} Bh, \quad (1)$$

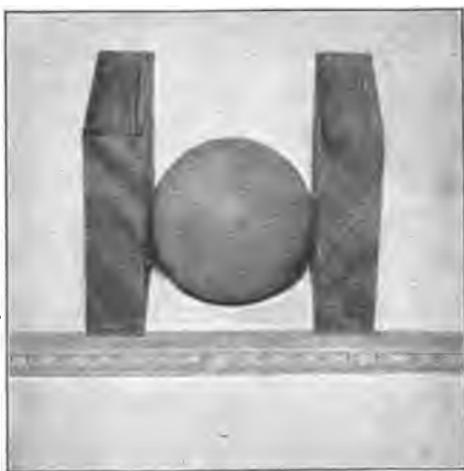
or,

$$V = \frac{1}{3} \pi R^2 h. \quad (2)$$

EXERCISES

1. Construct a paper cylinder which shall have a base and altitude equal to the base and altitude of the cone constructed for Ex. 3, page 179. Fill the cone with sawdust and pour into the cylinder as many times as is necessary to fill the cylinder. Does this prove equation (1) above?
2. Find the volume of the cone whose altitude is 6 in. and the radius of its base 5 in.
3. Find the volume of a cone which has the radius of its base 12.3 cm. and its altitude 14 cm.
4. The area of the base of a cone is 15 sq. in. and its volume is 35 cu. in. What is its altitude?
5. Tell how to find the altitude of a cone when its volume and the radius of its base are given. State this as an equation.
6. Use this equation to find the altitude of the cone whose volume is 56 cm³. and the area of its base 24 cm².
7. Find the altitude of the cone whose radius is to be 3 in. and volume is to equal that of a cone having an altitude 6 in. and the radius of its base 5 in.
8. The slant height of a cone is 14 in. and the altitude 9 in. Find the volume.

9. The slant height of a cone is 17 in. and the radius of its base is 9 in. Find its volume.
10. Find correct to two decimal places, the volume of the cone with a slant height of 8' and the radius of its base 3'.
11. Find the volume and the lateral area of the cone with an altitude of 6' and the radius of its base 2.5'.



192. Spheres.—A sphere is a solid bounded by a curved surface, all points of which are an equal distance from a point within, called its **centre**. The distance from any point on the surface to the centre is its **radius** and twice the radius is its **diameter**, just as in the circle. Any plane through a sphere cuts the sphere in a circle. If the plane passes through the centre of the sphere, the circle is then called a **great circle**.

EXERCISES

1. Give three illustrations of spheres.
2. Measure the diameter of three or four spheres as is suggested in the cut above.

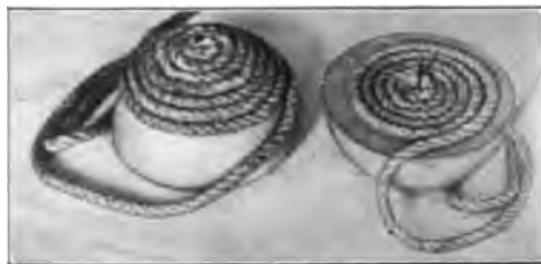
193. Areas of Spheres.—If a fairly heavy cord be wound around a hemisphere and also around a great circle having the same radius, it will be found that twice as much cord is needed to cover the hemisphere as the great circle. The surface of the hemisphere is, therefore, twice that of a great circle of the same radius. The total area of the sphere is then four times the area of the great circle, or

$$A = 4\pi R^2.$$

This fact is proved conclusively in geometry.

EXERCISES

1. Find the area of the sphere having a radius of 5".
2. Find the areas of the spheres measured in Ex. 2, page 181.
3. What is the area of the surface of a dome which is a hemisphere having a diameter of 28 ft.?
4. Carry out the experiment of winding a cord onto a hemisphere and a great circle.
5. A sphere of radius 3" can just be dropped into a cylinder which has an altitude of 6" and the radius of its base 3". Which has the larger area and how much?
6. Express the area of the earth in square miles by using 4000 mi. as the radius of the earth.
7. Tell how to find the radius of a sphere when its area is known. Express this as an equation.
8. The area of a sphere is 616 sq. in. Find its radius. See the above equation.



194. Volumes of Spheres.—Every sphere may be thought of as composed of a great number of very small equal pyramids. The volume of each little pyramid is $\frac{1}{3} Bh$ or $\frac{1}{3} BR$, since the height of the pyramid is the radius of the sphere. The volume of the sphere is the sum of the volumes of all these little pyramids. Their total volumes will be the radius of the sphere times the sum of the areas of their bases. Why? The sum of the areas of their bases is the total surface of the sphere.

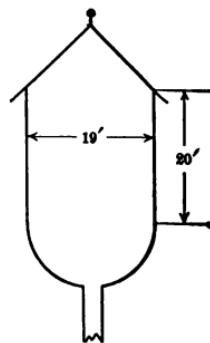
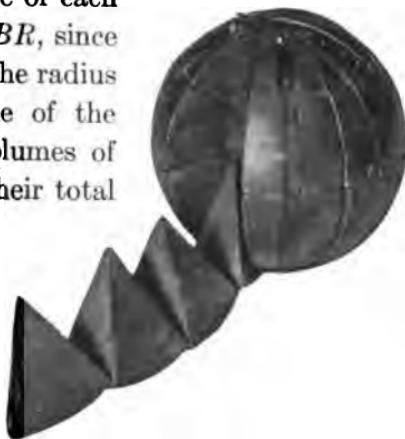
Why? Hence,

$$V = \frac{1}{3} R \times 4\pi R^2 \quad (1)$$

$$V = \frac{4}{3} \pi R^3. \quad (2) \text{ How?}$$

EXERCISES

1. Explain formula (2) fully.
2. Find the volume of a sphere whose radius is 3"; 6 cm.; .8 dm.; 1.2 ft.
3. Find the volume of the sphere which has a diameter of 6 in.; 10 cm.; 6 dm.; 1.4 ft.
4. From (2) express R in terms of V and π .
5. Find the radius of the sphere whose volume is $7\frac{1}{2}$ cm^3 ; $1437\frac{1}{3}$ cu. in.
6. The lower part of this tank is a hemisphere. The cylindrical part is 20 ft. high. Both have a radius $9\frac{1}{2}$ ft. Find its capacity in cubic feet and in gallons.
7. Play a number game, using measurements.



IX

DIRECT AND INDIRECT MEASUREMENTS

195. Direct Measurements.—Two kinds of measurements are made; direct and indirect. Direct measurements are such as (1) finding the length of a house by means of a yard-stick or a foot-ruler; (2) finding the weight of a sack of flour or the weight of a load of coal by the use of scales; (3) finding the volume of the milk in a pail or the volume of the berries in a basket by the use of a pint or a quart measure. All measures in which the result is in terms of the unit of measure needed are direct measurements. In most cases no computations are needed in direct measurements.

196. Indirect Measurements.—The plasterer measures the height and the length of the walls he is to plaster. From this he computes their areas. He measures a length when he wishes to find an area. In finding the number of bushels of wheat a load contains, the grain dealer weighs the load and after the grain has been unloaded, he weighs the wagon. The difference between these two weights he divides by 60, the number of pounds a bushel of wheat weighs. Why? The grain dealer weighs grain when he wishes to find its volume. These are only two of many indirect measurements. In fact, most of our measurements are indirect. All measurements in which the results are not found directly but are computed from other measurements are indirect. Areas and volumes have all been found from linear measurements and are therefore indirect.

EXERCISES

1. State which of the following measurements are direct and which are indirect:
 - a. Measuring the length of a room;
 - b. Weighing a can of milk to find the number of quarts of milk it contains;
 - c. Measuring milk with a quart measure;
 - d. Weighing out sugar on a pair of scales;
 - e. Measuring the length and width of a room to find the size of rug needed for it;
 - f. Weighing a load of coal and the empty wagon to find the number of tons of coal;
 - g. Taking the dimensions of a coal-bin to find the number of tons it will hold;
 - h. Measuring potatoes with a peck measure;
 - i. Measuring a hay-stack to find the number of tons of hay it contains.
2. Suggest three direct measurements other than those mentioned above. Why are they direct?
3. Suggest three indirect measurements other than those mentioned above. Why are they indirect?
4. Explain what measurements must be taken and how these are to be used in finding the volume and the surface of a sphere. Are these direct or indirect measurements?
5. A sphere is placed between two blocks as shown in the picture on page 181. The sides of the blocks nearest the sphere touch the meter stick at 4.6 cm. and 9.3 cm. from the same end of the meter stick. What is the radius of the sphere? Find its surface. Find its volume.
6. How far do you go in 10 steps? in 1 step? Use this measure to show the distance of 100 ft.; of 100 yd.; of 1 rd.

7. Select two points on the school ground or on the street and estimate the distance between them. Step off this distance to check your estimate. What is the per cent of error of your estimate? Make an estimate of three or four other distances in the same manner and check the estimate by stepping off the distance. Also find your per cent of error in each case.

8. The former army pace was $2\frac{1}{2}$ ft. How many inches is this? How many steps did a soldier take in marching a mile of 5280 ft.? 5 mi.? 15 Km.?

9. The new army step is 28 in. How many feet and fraction of a foot is this? How many steps will a soldier take in marching a mile? 5 mi.? 15 Km.?

10. If a soldier takes 126 of these steps per minute, how far does he go in a minute? in $\frac{1}{2}$ hr.? in 1 hr.? in 3 hr.?

11. An acre is 160 sq. rd. What length and width could a rectangle have so that its area would be an acre? Give at least three different suggestions.

12. Use the most convenient one of the different rectangles suggested for Ex. 11 and lay off a piece of ground which will be an acre.

13. How long are the sides of a square containing one acre?

14. Step off the length and the width of some city block and from this find its area in acres.

15. How many tons are there in a load of coal weighing 5230 lb. and the empty wagon weighing 1350 lb.? Is this a direct or an indirect measurement? Notice that you get weight by weighing.

16. How many bushels of oats are there in a load weighing 2940 lb. if the empty wagon weighs 1190 lb.? What is the cost of the oats at 57¢ per bushel?

197. Volumes of Irregular Solids. —

Two general methods are used in finding the volume of an irregular solid, as a stone. One is to weigh the body and divide that by the weight of a cubic inch or a cubic foot of the substance. A cubic inch of cork weighs about .15 oz. A piece of cork weighing $2\frac{1}{2}$ lb., that is 40 oz., would have a volume of $(40 \div .15)$ cu. in. How many cubic inches is this?



If the object is such that it will sink and not dissolve in water, the following is a good plan: Immerse the object in a cylinder of water and note how high the water rises. The volume of the object and the volume of the cylinder into which the water has risen compare how? For instance, a rock is immersed in a cylinder whose radius is 2" in which the water stands 8" above the bottom. After the rock is in the water the top level is 11" above the bottom. How high did the rock raise the water in the cylinder? What is the volume of the rock?

EXERCISES

1. A cubic foot of ice weighs 57.5 lb. What is the volume in cubic feet of a piece of ice weighing 230 lb.?
2. How many cubic feet of ice are there in a piece weighing 500 lb.? Give the result correct to two decimal places.
3. The ice compartment of a refrigerator is 9" x 15" x 28". Allowing one inch each way, how many cubic feet of ice will it hold? How much will the ice weigh?

4. • One cubic inch of silver weighs about .38 lb. What is the volume of a piece of silver weighing 5 lb. ? 12 lb. ?
5. How much would 8 cu. in. of silver weigh? How much would 6.5 cu. in. of silver weigh?
6. The silver dollar contains .84 oz. of silver. How many cubic inches of silver are there in the silver dollar?
7. Measure and weigh a common brick or rectangular piece of wood. Compute the weight of a cubic inch.
8. Measure just as accurately another piece of the same substance. From the weight of a cubic inch found in the last exercise compute how much this second piece should weigh. Next weigh it and compare this with the computed weight.
9. Tell how the weight of an object can be found if the weight of a cubic inch of the substance is known and its volume is also known.
10. If W is the weight of an object and w the weight of a cubic inch of the same substance, show that the answer to Ex. 9 can be stated by the equation,

$$W = wV. \quad (1)$$

11. From equation (1) the volume of an object can be found, providing its weight is known and the weight of a cubic inch or of a cubic foot of the substance is known.

$$V = \frac{W}{w}. \quad (2)$$

Explain this equation fully.

12. A lump of lead is immersed in a cylinder of water and raises the level of the water 8 cm. If the radius of the cylinder is 3 cm., what is the volume of the lead?
13. The base of a small rectangular tank is 6" x 8". It has water in it to a depth of 6". How much will the water rise if a common brick is immersed?

14. A cylinder has a radius of 2" and contains water to a depth of 8". A piece of granite is immersed and raises the top of the water to 11" above the bottom. Find the volume of the rock.
15. If a cubic inch of the granite used in Ex. 14 weighs 1.56 oz., what is the weight of the piece in Ex. 14?
16. A cylinder whose radius is 3" has .5" of water in it. A lead sphere with a radius of 2" is immersed in the cylinder. How much is the water in the cylinder raised?
17. A farmer piles on the ground his husked corn which falls in the form of a cone. This cone has a radius of 5' and an altitude of 4.5'. If $2\frac{1}{2}$ cu. ft. make a bushel, how many bushels are there in the pile?
18. A quantity of coal thrown into a heap settles into a cone which is 5' high and has a radius of 3'. Find the volume of the cone and the number of tons of coal it contains. 35 cu. ft. of the coal weigh a ton.
19. The floor of a coal-bin is a rectangle 8' by 19'. How many tons are there in the bin when the coal is 3.5' deep? 1' deep?
20. To what depth would the bin have to be filled in order to have 12 T. in the bin?
21. The floor of a coal-bin is in the form of a right triangle in which the perpendicular sides are 8' and 9.5'. How many tons will it hold, filled to a depth of 6'? of 1'?
22. Coal piled upon the ground takes a form nearly that of a cone. Tell what measurements you would take. Explain how you would use these to find the volume of the cone and from the volume, the number of tons it contains.
23. Find from measurements the contents of a heap forming a cone of sand, coal, or like substance.
24. Find accurately to two decimal places the radius of the sphere whose area is 352 sq. in.; 546 cm².

198. Hay Measurements.—The amount of hay in a stack is often determined from measurements by one of the following equations:

$$\text{Number of cubic feet} = L \times W \times \frac{1}{3} O. \quad (1)$$

$$\text{Number of cubic feet} = L(W + O)^2 \div 16. \quad (2)$$

$$\text{Number of cubic feet} = (R \div 4)^2 H. \quad (3)$$

$$\text{Number of cubic feet} = F \times O \times W \times L. \quad (4)$$

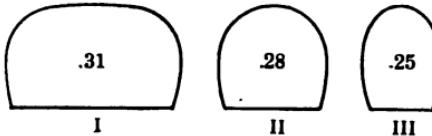
L is the length of the stack; H estimated height; W width; R distance around; O distance over; F a fraction varying with the shape of the stack as shown in the pictures.

512 cu. ft. = 1 T. (new hay).

422 cu. ft. = 1 T. (stack 5 mo. old).

342 cu. ft. = 1 T. (stack 1 yr. old).

216 cu. ft. = 1 T. (stack 3 or 4 yr. old).



EXERCISES

1. How many cubic feet of hay are in a stack like Fig. I, in which the distance around is 107' and the height 9'?
2. A stack like Fig. II is 38' long, 22' wide, and 39' over. Find by three methods and compare the number of tons of new hay it contains. What is your conclusion?
3. A stack like Fig. III is 47' long, 18' wide, and 37' over. Find by three processes the number of tons of old hay it contains and compare. What is your conclusion?
4. If hay is measured in this way in your community, find what equation is used. Try this formula in some of the above exercises and compare with the previous results.

199. Various Kinds of Meters.—Some words have more than one meaning. The word foot may refer either to a part of the body or to a measurement of 12 in. Similarly, the word **meter** has two meanings. We have already studied it as a unit of measure. We shall here consider it as an instrument used for measuring. Thus, there are water **meters**, electric **meters**, and gas **meters**. Sometimes meter forms just the last part of the word. The **cyclometer** is an instrument used to measure how far a rapidly moving body, such as an automobile, has gone. A picture is given on page 55. The **pedometer** is a small instrument carried in the pocket to tell the number of steps one has taken in going a certain distance. All measuring instruments called meters record as well as measure. We shall now study how to read and how to use these various forms of meters.

EXERCISES

1. The cyclometer of an automobile read 2646.1 mi. at the beginning of a trip and 3214.6 mi. at the end of the trip. What was the length of the trip in miles?
2. If R_b and R_e are the readings of the cyclometer on an automobile at the beginning and ending of a trip, express by an equation the distance, D , which has been travelled.
3. The cyclometer of an automobile read 814.3 mi. at the beginning of a trip and 903.6 mi. at the end of the trip. How far did the automobile go? At what rate per hour was the trip made if it required 4 hr.?
4. Use the same letters as in Ex. 2 and with the same meaning express as an equation the velocity, v , of travelling the distance in h hours.
5. Play a number game, using percentage.

6. Find the rate of speed of an automobile for which the cyclometer reads 1673.4 mi. and 1936.8 mi. at the beginning and end of a trip. It required 9 hr. 10 min. of actual running time to make the trip.
7. Henry starts out in the morning with the cyclometer of his bicycle reading 479.3 mi. After riding 5 hr. during the day he finds that the cyclometer reads 523.7 mi. What has been his average speed?
8. An automobile averages 25 mi. per hour when running. If the cyclometer read 1357.6 mi. when beginning a trip, what should it read after 4 hr.? after 5 hr.? after 7 hr.?
9. The cyclometer of John's bicycle reads 10 % high. If it read 156.4 mi. at the beginning of a trip and 184.2 mi. at the end of the trip, how many miles did John ride?
10. The cyclometer on William's bicycle reads 15 % low. If it read 1274.2 mi. at the beginning of a trip and 1312.8 mi. at the end of the trip, how many miles did William ride?
11. Threshing-machines record the number of bushels of grain which have been threshed by them. A threshing-machine records 5369 bu. at the beginning of threshing a 60-A. field of wheat and records 6253 bu. at the end of the work. How many bushels have been threshed? What was the yield per acre? For how much did the wheat sell at \$ 1.87 $\frac{1}{2}$ per bushel?
12. A threshing-machine records 2716 bu. when work is begun and 5432 bu. when work is completed in threshing a 120-A. field of oats. How many bushels were threshed? What was the yield per acre? What were the oats worth at 69 $\frac{1}{2}$ ¢ per bushel?

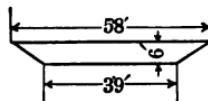
13. A pedometer records one for every step the person carrying it takes. In a boy scout's tramp, where the average step is 2', Harry carries a pedometer that has a first reading 2957 and a last reading 3245. How far did the boys go?

14. A soldier carried a pedometer which read 18,349 at the beginning of a march and 33,125 at the end of the march. At 28 in. to the step, how long a march had he taken?

15. A line of soldiers $\frac{3}{4}$ mi. long passes a reviewing-stand. How far have the first soldiers gone when the last soldiers are opposite the reviewing-stand? If this has taken 15 min., how long will it take the soldiers to march a mile? How many miles will they march per hour?

16. John threw a stick upon the water in a stream and found that it floated 900' in 5 min. How many miles is that per hour? How many feet is it per minute?

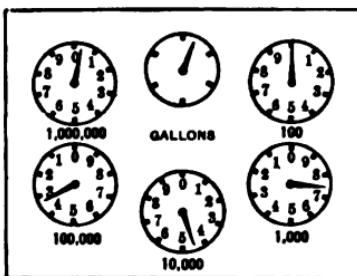
17. If the stream has a cross-section, as shown in the picture, what is the area of the cross-section?



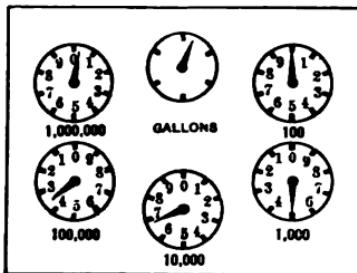
18. Every minute the water moves 180'. That is, the water from a prism whose bases have the areas you have just computed and the length of 180', flows past some one point every minute. How many cubic feet is this? What is therefore the rate of flow past any place in the stream per minute? per second? per hour?

19. A water-pipe has a radius of 1''. What is the area of a cross-section? If the water flows along at the rate of 3' per second, how much water flows out of the pipe in a second? in a minute? in an hour?

20. What is the rate of flow per minute of water from a pipe 1.5" in diameter, which flows along in the pipe at the rate of 2' 6" per second?



I



II

200. Water Meters.—Water from a city water-works is often measured by meters and sold by the 1000 gal. The top dial is used only in testing the meter. Below each dial is stated the number of gallons consumed when its hand has made one complete revolution. Note that the hands of any dial and the next rotate in reverse directions. The difference in readings at the end of the past and of the present month gives the amount of water consumed this month. Water meters are read only to the even hundred gallons. Meter I records 34,700 gal.

EXERCISES

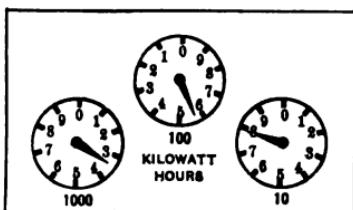
1. Read the above meters.
2. The readings of a water meter at the end of March was 134,000 gal. and at the end of April it was 142,000 gal. How much water was used during April?
3. At the end of May the same meter recorded 151,800 gal. How many gallons were used during May? How many cubic feet is this?
4. In a factory the reading of the water meter at the end of July was 567,200 gal. and at the end of August it was 732,700 gal. How many gallons of water were used during August?

CITY WATER Emporia, Kansas.	January 1, 1919. Account for December, 1918.
State of Meter.....	235600 Street State
Former Reading.....	219800 Number 1119
Consumed.....	15800 Total bill..... \$ 3.66
Rate per 1,000 gallons:	
First 10,000....	.25 Next 25,000.... .15
Next 10,000....	.20 Next 35,000.... .14
Next 10,000....	.18 Next 50,000.... .13
Next 10,000....	.16 Next 50,000.... .12

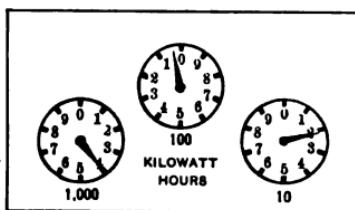
201. Water Bills.—Monthly water bills are made out like the one here shown. Rates, as those given below the bill, are generally printed on the opposite side.

EXERCISES

1. Check the above bill to find if it is correct.
2. Suppose that the water meter for a house was, as shown in the pictures on the opposite page, at the end of October and November. Make out a bill for November.
3. At the rate printed below the above bill, find the cost of the water used during August by the factory for which the meter readings are given in Ex. 4, page 194.
4. If you have a water meter at home or at the school, read it and record your readings at intervals of a week for a month. How many gallons were used each week? in the month? Find the cost at the rates in your city.
5. How many families are there in a city of 10,000, estimating 5 per family? Find the monthly cost of water per family and for the city at the above rate if 4500 gal. is the average per family per month. How deep would the water fill a reservoir the size of a city block, 250' x 300'?
6. Find the data to answer these questions for your city.



I



II

202. Electric Meters.—Electricity for lighting, for running machinery and, in some communities, for cooking is sold by an electric unit called the **watt-hour**. Incandescent lamps are marked 15 W, 25 W, and so on. They consume 15 watt-hours, 25 watt-hours, and so on, of electricity each hour that they are burning. Electric appliances sometimes have marked upon them the voltage, V , and the current, C (or i), which they take. The product of these two numbers is the number of watt-hours the appliance will consume per hour. Electricity is generally sold by the kilowatt-hour ($K.W.$), that is, by the 1000 watt-hours.

EXERCISES

1. Give the reading of the above electric meters.
2. If the reading of the electric meter at the end of May, is 386 $K.W.$ and at the end of June is 412 $K.W.$, how many kilowatt-hours of electricity were used during June?
3. How many watt-hours are used in burning five 40-watt lamps and three 60-watt lamps each for $3\frac{1}{2}$ hr.? How many kilowatt-hours is this?
4. How many watt-hours does Henry waste by forgetting to turn out two 25-watt lamps in the basement if they burn 8 hr.? Express this in kilowatt-hours.
5. At 11¢ per kilowatt-hour, how much does Henry waste by forgetting these lamps three nights?

6. An electric iron is marked 110 v. and 6.35 i. What is the cost per hour of using the iron at 12¢ per kilowatt-hour?

7. How many kilowatt-hours are used in running an electric vacuum cleaner an hour, if it takes 110 v. and 1.4 i.?

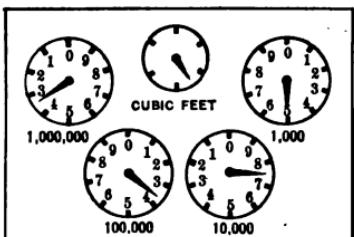
To THE KANSAS ELECTRIC UTILITIES CO., Dr. <i>Bring This Card With You.</i>		Book No. 7
		Cust. No. 314
Present Reading	256 K.W.	
Former Reading	229 K.W.	
Amount Consumed	27 Rate	
LIGHTING RATE:	Amount	2.70
No Discount		
0 to 500 K.W., 10c.	Balance	
Excess of 500 K.W., 8c.		
POWER RATE:		
No Discount		
First 100 K.W., .08.		
Next 100 K.W., .07.		
Next 1300 K.W., .06.	Total	2.70
Over 1500 K.W., .05.		Total

8. Check the above bill to see if it is correct.

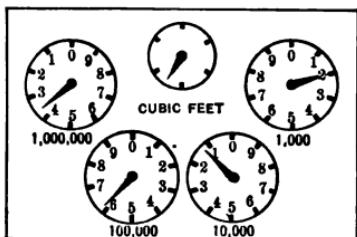
9. Make out an electric bill like the above for the month of December, if the reading of the meter at the end of November was 748 K.W. and at the end of December 815 K.W.

10. Fill out a similar bill for the month of February if the reading at the end of January was 372 K.W. and at the end of February 458 K.W.

11. If you have an electric meter at the school, or at your home, read it and record the result weekly for a month. How many kilowatt-hours were used each week and during the month? If you found variations for the weeks, try to give reasons for these variations.



I



II

203. Gas Meters.—Gas is used for cooking and for lighting. It is sold by the cubic foot in much the same way as water is sold by the gallon. The gas meter is read only to the nearest 100 cu. ft. The price is a certain amount per 1000 cu. ft. As with electricity a discount is generally given for prompt payment.

EXERCISES

- If the above give the meter readings for the end of November and December, how many cubic feet of gas were used during December?
- The gas meter reading at the end of May was 128,400 cu. ft. and at the end of June it was 141,200 cu. ft. How many cubic feet were used during June?
- Suppose that the price of gas in the town where the above reading was taken was \$ 1.25 per *M*, with a discount of 20 % if the bill was paid on or before the 10th of the month. For how much could the bill be paid June 5th? June 9th? June 10th? June 16th?
- If there is a gas meter at your school or at your home, read it at a certain time on a certain day each week for a month and record the reading each time. How much gas has been consumed each week and during the month? Try to account for any great variation from week to week. Find cost for each week and the month at the rate in your town.

5. Check the following bill to find if it is correct. Correct any errors.
6. Make out a bill similar to that below, using the following readings: end of March 345,200 cu. ft. and end of April 351,100 cu. ft.
7. At the rate given on the bill below, find the cost of the gas for a manufacturing plant for which the meter read

Emporia, Kansas, FEB. 1. 1919					
To EMPORIA GAS CO., Dr.					
PHONE 286	519 Merchant Street				
Reading taken Jan..... 1919	3	5	2	3	0 0
Reading taken Dec..... 1918	3	4	6	8	0 0
Gas Consumed		5	5	0	0
Cu. ft. at Sched. Rate	\$	5.00			
Discount.....	\$.80			
Net	\$	7.20			
Gas Arrears.....	\$				
Gas Arrears.....	\$				
Total	\$	7.20			

Schedule of Gas Rates

Gross, \$1.70; net, \$1.60 per M for 1st and 2nd 1,000 cubic feet.
 Gross, \$1.50; net \$1.40 per M for next 3,000 cubic feet.
 Gross, \$1.30; net \$1.20 per M for next 5,000 cubic feet.
 Gross, \$1.20; net 1.10 per M for next 10,000 cubic feet.
 Gross, \$1.20; net 1.00 per M for next 5,000 cubic feet.
 Gross, \$1.00; net 75¢ per M for over 25,000 cubic feet.
 Minimum bill, 50 cents per month.

143,500 cu. ft. at the beginning of a certain month and 151,300 cu. ft. at the end of the same month.

8. Mrs. Carenot uses her oven which consumes 45 cu. ft. of gas per hour at an average of $\frac{3}{4}$ hr. per day. During two-thirds of this time she could use a detachable oven placed over a burner consuming only 15 cu. ft. of gas per hour. At \$1.10 per 1000 cu. ft., what would she save in using the detachable oven per week? per month? per year? How long would it take to save the price of the detachable oven which is \$2?

9. What is the price of 4100 cu. ft. of gas at the rate given on the bill above?
10. Play a number game, using common fractions.

X

SIGNED NUMBERS

204. Meaning of Signed Numbers.—Numbers giving the amount of money earned and the amount of money spent are **opposite numbers**; distance north and distance south are also **opposite numbers**. Pairs of two such numbers which are opposite in meaning occur continually in higher mathematics, in science, in the various industries, and in business. One set of the numbers of such pairs is called **positive** and is preceded by the sign +. The other set of the numbers of such pairs is called **negative** and is preceded by the sign -. Thus, \$5 earned would be represented by + \$5, while \$5 spent would be represented by - \$5. Suppose that we decide to call north positive and south negative. Then + 4 mi. would be 4 mi. north and - 7 mi. would be 7 mi. south. John and Henry belong to a baseball team whose players have an average weight of 108 lb. If John weighs 113 lb., the difference between his weight and the average weight is + 5 lb. If Henry weighs 102 lb., the difference between his weight and the average weight is - 6 lb. One is **above** and one is **below** the average weight.

Because of the use of the signs + and -, these numbers are called **signed numbers**.

In a junior high school number game each correct result was marked + 10, and each error and unsolved exercise - 10. Can you explain this use of signed numbers? It will be shown in Art. 206 how to combine such signed marks.

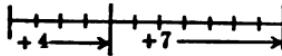
EXERCISES

Show that the following 1-10 Exs. contain pairs of opposite numbers:

1. Going west and going east.
2. Borrowing money and paying back money.
3. Buying and selling.
4. Selling at a profit and selling at a loss.
5. Going up and going down a hill.
6. Making scores in a baseball game and having the other team make scores.
7. Making put outs and assists, and making errors in a baseball game.
8. Going north and going south.
9. Putting money in the savings bank and drawing money out of the savings bank.
10. Water flowing into a cistern and water pumped out of the cistern.
11. Give three other illustrations of signed numbers.
12. If 9 mi. east is represented by + 9 mi., how would you represent 13 mi. west? 5 mi. east? 4.3 mi. west?
13. If paying \$ 7 on a debt is represented by + \$ 7, how would you represent borrowing \$ 32? paying \$ 32? paying \$ 12?
14. Express as signed numbers: (a) drawing a sled 85 ft. up a hill; (b) sliding 63 ft. down a hill.
15. Express as signed numbers: (a) depositing \$3.25 in the savings bank; (b) drawing \$1.35 from the bank.
16. State as signed numbers the number of scores your team is ahead or behind: (a) when you have 6 and the other team 4 scores; (b) when you have 5 and the other team 8 scores.

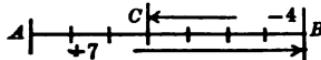
205. Adding Signed Numbers.—Adding signed numbers means combining them into one signed number.

Suppose that motion east is called $+$ and motion west is $-$. Then going $+ 4$ mi. means what? Going $+ 4$ mi. and $+ 7$ mi. means going 11 mi. east. Why? This can be expressed by an equation, thus:



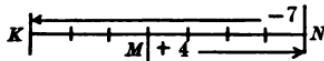
$$+ 4 \text{ mi.} + (+ 7 \text{ mi.}) = + 11 \text{ mi.}$$

Going $+ 7$ mi. from A to B , and then $- 4$ mi., B to C , is the same as going $+ 3$ mi. from the starting point A to C .



$$+ 7 \text{ mi.} + (- 4 \text{ mi.}) = + 3 \text{ mi.}$$

Again, going $+ 4$ mi. from M to N , and then $- 7$ mi. N to K , is the same as going $- 3$ mi. from the starting point M to K .



$$+ 4 \text{ mi.} + (- 7 \text{ mi.}) = - 3 \text{ mi.}$$

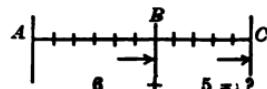
The truth of the last result is seen from the figure or from the values written thus:

$$+ 4 \text{ mi.} + (- 4 \text{ mi.}) + (- 3 \text{ mi.}) = - 3 \text{ mi.},$$

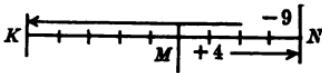
because the $+ 4$ mi. and the $- 4$ mi. being opposite destroy each other.

EXERCISES

- Let motion to the right be $+$ and to the left $-$. John went 6 steps from A to B and then 5 steps from B to C . Find the sum of these distances.



2. Harry went 4 steps to the right, M to N . He then turned around and went 9 steps to the left, N to K . Find the sum of these two signed distances.



Express the following motions as signed numbers and find their sum:

3. Eight steps to the right and 5 steps to the right.
4. Ten steps to the right and 17 steps to the left.
5. Nine steps to the left and 3 steps to the left.
6. Eleven steps to the right and 5 steps to the left.
7. Make up four other problems on steps and solve.

If + means distance north and - means distance south, state the distance from the starting point that each of the following represents:

8. $+ 5 \text{ mi.} + (+ 7 \text{ mi.})$
9. $+ 6 \text{ mi.} + (- 4 \text{ mi.})$
10. $+ 5 \text{ rd.} + (- 8 \text{ rd.})$
11. $+ 19 \text{ mi.} + (- 23 \text{ mi.})$
12. $- 8 \text{ mi.} + (+ 9 \text{ mi.})$
13. $+ 13 \text{ rd.} + (- 19 \text{ yd.})$
14. $- 3 \text{ mi.} + (- 4 \text{ mi.})$
15. $+ 2 \text{ mi.} + (- 7 \text{ mi.})$
16. $- 8 \text{ mi.} + (- 5 \text{ mi.})$
17. $- 5 \text{ yd.} + (+ 27 \text{ ft.})$
18. An automobile went 216 mi. east and returned 185 mi. Express these distances as signed numbers and find the position of the automobile.
19. One week a merchant drew \$ 468 out of his bank and deposited \$ 572. Express this by signed numbers and combine them so as to show his total transactions with the bank.
20. Tell how to find the sum of two signed numbers.
21. Play a number game on combining two signed numbers.

206. Addition of Several Signed Numbers.—If a merchant sells goods for \$ 85, \$ 137, and \$ 106 on three days and buys goods for \$ 65, \$ 43, and \$ 157 on the same days, his statement of sales and purchases would be:

$$\begin{aligned} + \$ 85 + (+ \$ 137) + (+ \$ 106) + (- \$ 65) + (- \$ 43) \\ + (- \$ 157) = + \$ 63, \end{aligned}$$

or his sales would be \$ 63 above the purchases. If his sales had been the same but his purchases had been \$ 74, \$ 196, and \$ 138, then the statement would have been:

$$\begin{aligned} + \$ 85 + (+ \$ 137) + (+ \$ 106) + (- \$ 74) + \\ (- \$ 196) + (- \$ 138) = - \$ 80, \end{aligned}$$

or the purchases would have been \$ 80 more than the sales.

In adding several signed numbers, find the sum of the positive and of the negative numbers separately. Then take the difference between the two and prefix the sign of the larger number.

Literal signed numbers are added in the same way. Thus,

$$+ 5ab^2 + (- 3ab^2) + 9ab^2 + (- 4ab^2) = + 7ab^2,$$

and

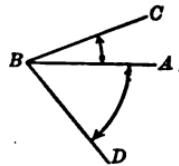
$$+ 3mn^3 + (- 7mn^3) + (+ 5mn^3) + (- 12mn^3) = - 11mn^3.$$

EXERCISES

1. During one week a merchant deposited in the bank \$ 135, \$ 285, \$ 95 and withdrew \$ 25, \$ 295, \$ 35. Express his dealings with the bank as the sum of signed numbers and give the result as a signed number.

2. Express as signed numbers a loss of \$ 1560, a gain of \$ 450, and a gain of \$ 715. Find the sum of these signed numbers and interpret the result.

In rotating a line about B so as to form an angle with AB , as angle ABC , the angle is called positive, if the line is rotated counter clock-wise and negative if the line is rotated clock-wise. Thus the angle ABC is positive and the angle ABD is negative.



3. Draw the following angles: $+ 45^\circ$; $- 60^\circ$; $+ 120^\circ$; $+ 35^\circ$; $- 145^\circ$; $+ 90^\circ$; $- 270^\circ$; $- 180^\circ$; $+ 180^\circ$.

Find the sum of the angles in the following Exs. 4-9 and locate the resulting angle:

4. $+ 45^\circ + (- 30^\circ)$	7. $- 90^\circ + (+ 45^\circ)$
5. $+ 30^\circ + (- 60^\circ)$	8. $- 60^\circ + (+ 30^\circ)$
6. $+ 90^\circ + (- 60^\circ)$	9. $+ 45^\circ + (- 60^\circ)$

10. A man who can row 5 mi. per hour in still water is rowing up-stream against a current of 2 mi. per hour. Show that the man's motion is

$$+ 5 \text{ mi.} + (- 2 \text{ mi.}) = + 3 \text{ mi.}$$

What does $+ 3$ mi. mean?

11. A boy can row 2 mi. per hour in still water. If the current is 4 mi. per hour, what will be his up-stream motion? Another boy comes in to help and increases the rate 3 mi. per hour. Express the motion of the boat with the two boys rowing and find the resulting motion.

Find the sums of the following signed numbers:

12. $+ 4mn^2$	13. $- 5pr^3$	14. $+ 8hk$	15. $+ 7s^3t^3$
$- 6mn^2$	$+ 8pr^3$	$- 5hk$	$- 3s^3t^3$
$+ 9mn^2$	$- 3pr^3$	$+ 6hk$	$+ 8s^3t^3$
$- 5mn^2$	$- 8pr^3$	$+ 3hk$	$- 9s^3t^3$
$+ 3mn^2$	$- 2pr^3$	$- 2hk$	$+ 3s^3t^3$

16. In a number game Harry had a score $+ 10$, $+ 10$, $- 10$, $+ 10$, $+ 10$, $- 10$, $+ 10$, $- 10$, $+ 10$. Find his total score.

17. Play a number game in which you keep the score by signed numbers as suggested on page 200.

207. Subtraction of Signed Numbers.—Subtraction of signed numbers is the reverse—the opposite—of addition, just as subtraction is the reverse of addition with all numbers. Thus, $+ 5$ less $+ 3$ means to find the number which added to $+ 3$ gives $+ 5$. What is this number? Similarly, $+ 6$ less $- 4$ means to find the number which added to $- 4$ gives $+ 6$. What is this number? How much is $- 7$ less $- 5$? That is, how much must be added to $- 5$ in order to get $- 7$? What is this? Notice that if the sign before the signed number in the subtrahend be changed and the numbers be then added, the result will be the subtraction asked for. Do not make these changes in sign on paper; merely think them changed.

By using signed numbers, a number may be subtracted from a smaller one. Thus, $+ 8 - (+ 9)$ means finding the number which added to $+ 9$ gives $+ 8$. This is clearly $- 1$.

EXERCISES

1. How much must be added to 15 bu. to get 25 bu.?
2. Find the difference between 15 bu. and 25 bu.
3. From $+ 17$ take $+ 12$. What added to $+ 12$ gives $+ 17$?
4. From $+ 13$ take $+ 16$. What added to $+ 16$ gives $+ 13$?
5. From $+ 8$ take $- 7$. What added to $- 7$ gives $+ 8$?
6. From $- 11$ take $- 6$. What added to $- 6$ gives $- 11$?
7. From $- 13$ take $+ 9$. What added to $+ 9$ gives $- 13$?
8. From $- 4$ take $- 15$. What added to $- 15$ gives $- 4$?

Make the following subtractions:

- | | |
|--------------------|-------------------|
| 9. $+ 6 - (+ 5)$ | 12. $- 6 - (+ 8)$ |
| 10. $- 4 - (- 3)$ | 13. $- 8 - (+ 6)$ |
| 11. $+ 13 - (- 7)$ | 14. $- 6 - (- 7)$ |

$$\begin{array}{r} \text{15. } + 13TR \\ + 8TR \\ \hline \end{array}$$

$$\begin{array}{r} \text{19. } + 56xc^2 \\ + 69xc^2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{23. } - 63t^3k^3 \\ + 9t^3k^3 \\ \hline \end{array}$$

$$\begin{array}{r} \text{16. } + 413m^2n \\ + 248m^2n \\ \hline \end{array}$$

$$\begin{array}{r} \text{20. } - 18y^3v \\ - 26y^3v \\ \hline \end{array}$$

$$\begin{array}{r} \text{24. } - 11fd^3 \\ + 9fd^3 \\ \hline \end{array}$$

$$\begin{array}{r} \text{17. } - 9g^3k^2 \\ + 3g^3k^2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{21. } + 9qr^2 \\ - 3qr^2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{25. } - 91r^2y \\ + 75r^2y \\ \hline \end{array}$$

$$\begin{array}{r} \text{18. } - 17a^2b \\ - 8a^2b \\ \hline \end{array}$$

$$\begin{array}{r} \text{22. } - 78p^2q \\ - 45p^2q \\ \hline \end{array}$$

$$\begin{array}{r} \text{26. } - 63m^5n^6 \\ - 97m^5n^6 \\ \hline \end{array}$$

27. Add $+ 15$ and $- 36$ and from their sum take $+ 29$.

28. From the sum of 45 and $- 17$ take $- 32$.

29. Subtract $- 258$ from the sum of $+ 272$ and $- 396$.

30. Take $- 597$ from the sum of $+ 739$ and $- 127$.

31. From the sum of $+ 572$ and $- 317$ take the sum of $- 184$ and $+ 96$.

32. If 1919 A.D. is written $+ 1919$, how should 323 B.C., the year Alexander the Great died, be written? How many years ago did Alexander the Great die?

33. A merchant's profits during four months are represented by $+ \$459$, $- \$630$, $+ \$125$, $- \$75$. Find their sum and explain its meaning.

34. The following table gives the average height in inches of boys and girls of the ages stated:

Girls

Boys

9 yr.— 48.0	12 yr.— 54.6
---------------	----------------

9 yr.— 48.8	12 yr.— 54.0
---------------	----------------

10 yr.— 50.2	13 yr.— 56.9
----------------	----------------

10 yr.— 50.8	13 yr.— 56.2
----------------	----------------

11 yr.— 52.5	14 yr.— 58.7
----------------	----------------

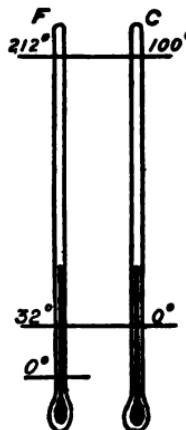
11 yr.— 52.5	14 yr.— 58.4
----------------	----------------

Measure your height accurately. State as a signed number, the difference between your height and the average height for your age.

208. Thermometry.—All are familiar with the mercurial or spirit thermometers that are used to tell the amount of warmth possessed by any substance, as the air in a room. The instrument consists of a sealed glass tube with a bulb at one end. The bulb and part of the tube are filled with mercury or spirits. Almost all substances expand when heated. Mercury expands faster than glass when both are heated to the same temperature. Hence, the mercury in the thermometer tube rises when the thermometer is heated.

Thermometers are made with two kinds of scales as shown in the picture. The more common Fahrenheit thermometer is the one found in most of our homes. The Centigrade thermometer, used by scientific men, is based upon the decimal system and is by far the simpler. Each scale has a zero point from which readings are made. It is customary to write $+ 12^\circ$ for 12° above zero and $- 12^\circ$ for 12° below zero on either scale. To express that a thermometer standing at $+ 25^\circ$ has fallen 34° would be,

$$+ 25^\circ + (- 34^\circ) = - 9^\circ.$$

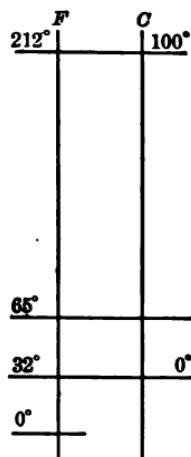


EXERCISES

- If there is a Fahrenheit thermometer in the school, point out the following points, or draw a scale to show them: $+ 65^\circ$; $+ 85^\circ$; $- 50^\circ$; $- 36^\circ$; $+ 19^\circ$; $- 32^\circ$.
- Which of the readings in Ex. 1 would the thermometer indicate when it is warm? when it is cold?
- How high does the thermometer read in summer where you live? Find this on the scale.

4. How low does the thermometer read in winter where you live? Find this on the scale.
5. Read the thermometer at your home or at the school for to-day and for each day at the same time for a week and record the readings. What changes did you find?
6. Repeat the above experiment for four weeks. Be careful to record your readings accurately and clearly so that they can be preserved for the work next year.
7. Estimate the readings of the thermometers in the picture on page 208.
8. Draw the Centigrade scale upon the board and point out the following points: $+ 17^\circ$; $- 8^\circ$; $+ 29^\circ$; $- 11^\circ$.
9. How far are the readings given in Ex. 1 from freezing? Express these as signed numbers.
10. How far from freezing are the readings given in Ex. 7? Express these as signed numbers.
11. What is the change in degrees of either the Fahrenheit or the Centigrade scale from $+ 12^\circ$ to $+ 25^\circ$? from $+ 36^\circ$ to $- 13^\circ$? from $- 18^\circ$ to $- 37^\circ$? from $- 7^\circ$ to $- 3^\circ$? from $- 16^\circ$ to $+ 48^\circ$? from $+ 58^\circ$ to $- 17^\circ$?
12. What will be the reading of either scale after a rise of 15° from $+ 37^\circ$? a drop of 32° from $+ 28^\circ$? a rise of 37° from $- 18^\circ$?
13. The first reading of a thermometer is $+ 56^\circ$. Then there is a drop of 15° ; a rise of 24° ; a drop of 19° ; a rise of 8° . Give the statement in signed numbers and find the final reading of the thermometer.
14. Repeat Ex. 11 with other readings.
15. Repeat Ex. 12 with other readings.
16. Repeat Ex. 13 with other readings.

209. Changing from One Thermometer Scale to the Other.—Recipes and formulas are often given for one thermometer scale while we may have only the other thermometer. In all such cases it becomes necessary to change the readings of the thermometer that we possess to that of the other scale, or to change the formula to agree with our thermometer. It will be noticed in the accompanying picture that the 180° between the freezing and the boiling points on the Fahrenheit scale correspond to 100° on the Centigrade scale. Hence $1^{\circ} \text{ C} = 1.8^{\circ} \text{ F}$. When changing a reading from one scale to the other, first draw two vertical lines and put in the constant points as here shown. Next put in the reading that is given and make a mark on the other scale to show where the reading will come on that scale, as in the picture. It is the number of degrees above or below zero that is required. The reading recorded in the picture, $+ 65^{\circ} \text{ F}$ is 33° above freezing on the Fahrenheit scale. These $+ 33^{\circ}$ on the Fahrenheit scale correspond to $33^{\circ} \div 1.8^{\circ}$, or 18.33° on the Centigrade scale. Therefore when the Fahrenheit thermometer reads $+ 65^{\circ}$ the Centigrade thermometer reads $+ 18.33^{\circ}$. Explain.



Suppose that $+ 15^{\circ} \text{ C}$ is to be changed to F. The $+ 15^{\circ}$ are 15° above freezing on the Centigrade scale. These 15° C equal $15^{\circ} \times 1.8$, or 27° F . The reading on the Fahrenheit scale is hence 27° above freezing, or $+ 27^{\circ} + (+ 32^{\circ}) = + 59^{\circ} \text{ F}$.

To change $- 4^{\circ} \text{ F}$ to C first draw the scale and locate the point. How far is this point below freezing on the Fahrenheit scale? How many degrees does this correspond

to on the Centigrade scale? Hence, -4° on the Fahrenheit thermometer is equivalent to what reading on the Centigrade thermometer?

EXERCISES

Draw the Fahrenheit and the Centigrade scales and show the following readings. Also find the corresponding reading on the other thermometer.

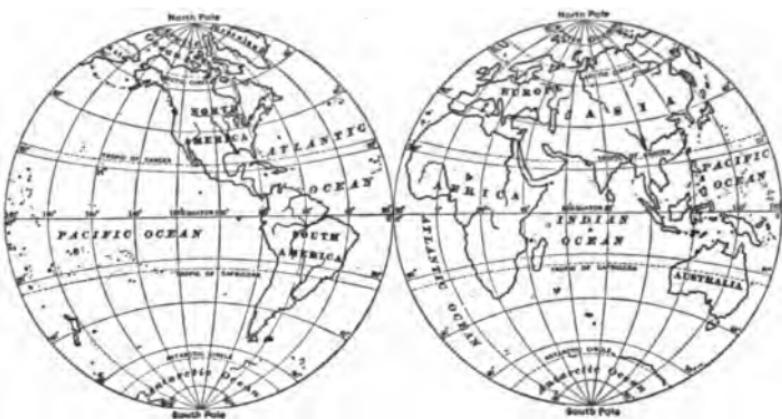
- | | | |
|---------------------|---------------------|--------------------|
| 1. $+ 45^{\circ}$ C | 4. $- 13^{\circ}$ C | 7. $- 4^{\circ}$ C |
| 2. $+ 68^{\circ}$ F | 5. $- 22^{\circ}$ F | 8. $- 8^{\circ}$ F |
| 3. $- 4^{\circ}$ F | 6. $+ 48^{\circ}$ C | 9. $- 7^{\circ}$ C |

10. Different substances boil at different temperatures, as given in the accompanying table:

Alcohol	$+ 78^{\circ}$ C
Ether	$+ 38.5^{\circ}$ C
Ammonia	$- 33^{\circ}$ C
Carbon dioxide	$- 80^{\circ}$ C
Liquid air	$- 182^{\circ}$ C

Express as a signed number how much the boiling point of these various substances is above or below the boiling point of water, which is 100° C.

11. Change any two readings in Ex. 10 to F.
12. Five boys in a class found by measurement that they were above or below the average heights given on page 207 by the following amounts: 1.8 in. above; 1.3 in. below; 0.8 in. below; 0.6 in. above; 1.2 in. above. Express each as a signed number and find their sum. What does the sum tell?
13. Apply Ex. 12 to the boys of your class.
14. Apply Ex. 12 to the girls of your class.
15. Play a number game, using percentage. Score the work as in the number game told of on page 200.



210. Meridians of Longitude.—Very long distances on the earth's surface are expressed in degrees by means of imaginary circles, as pictured in the hemispheres above. The great circles passing through the poles are called **meridian circles**, or merely **meridians**. What are great circles? In speaking of a meridian we shall refer only to half of the circle, the part reaching from one pole to the other pole as shown in the map above. Midway between the poles the **equator**, another great circle, encompasses the earth. There are 360 meridians crossing the equator 1° apart. The meridian which passes through Greenwich, England, has been chosen as the **prime meridian**, that is, the one from which we number. Along the equator the meridians are marked both ways, east and west, for 180° . Here the distance the meridian through a place is east or west of the prime meridian, can be read in degrees. This distance in degrees is called the **longitude** of a place, west longitude being + and east longitude being -. Thus, the longitude of New York is $+74^\circ\ 0' 3''$, while the longitude of Petrograd is $-30^\circ\ 19'$.

211. Parallels of Latitude.—Parallel to the equator, north and south, are drawn small circles called **parallels of latitude**, or simply **parallels**. The principal parallels cross each meridian one degree apart. This gives in degrees the distance of a place north or south of the equator. This distance is called **latitude** and is marked + for north and — for south latitude. The latitude of New York is $+ 40^{\circ} 42'$, of San Diego $+ 32^{\circ} 44' 41''$, while that of Rio de Janerio is $- 22^{\circ} 54'$.

A glance at the map on the opposite page will show that 5° of longitude near the equator are much longer in miles than 5° of longitude near the poles.

The difference in latitude or in longitude between two places is the difference between two signed numbers. Thus, the difference in longitude between Ann Arbor, Mich., and Chicago, Ill., is $+ 87^{\circ} 36' 42'' - (+ 83^{\circ} 43' 48'')$, or $3^{\circ} 52' 54''$. Between Chicago, Ill., and Paris, France, the difference in longitude is $87^{\circ} 36' 42'' - (- 2^{\circ} 20' 15'')$, or $89^{\circ} 56' 57''$.

EXERCISES

- 1.** Find the difference in longitude between New York and Petrograd; Paris and Petrograd; New York and Chicago; New York and Paris.
- 2.** Find the difference in latitude between New York and Rio de Janerio; New York and San Diego; San Diego and Rio de Janerio.
- 3.** Find on the map of the United States, in your geography, two cities far apart that differ little in longitude; two cities far apart that differ little in latitude. Why is this so?

212. Relation Between Longitude and Time.—Place a lighted candle or an electric light near a globe in a room in which the curtains have been lowered. Turning the globe will show clearly the coming of day and the coming of night over the earth's surface. Noon will be at that part of the earth's surface directly in front of the light. All points on any meridian will have noon at the same instant, while all points on a meridian west will have their noon later and all points on a meridian east will have had their noon already.

As the earth turns once about its axis daily, every point upon the earth turns through a complete circle, that is, through 360° , every 24 hours. In one hour every point on the earth's surface turns through $\frac{1}{24}$ of 360° , or 15° . That is, a point 15° west of where you live will have its noon one hour later than you. Why? A difference in longitude between two places of 15° equals $15 \times 60'$. This corresponds to a difference of 1 hr., or of 60 min., in time between two places. Therefore a difference of 1 min. in time corresponds to $\frac{1}{60}$ of $15 \times 60'$, or $15'$ of arc of a circle. Similarly, a difference of 1 sec. in time corresponds to a difference of $15''$ of arc of a circle. Carry out the computations for this. Therefore, the **number** expressing the difference in longitude between two places in degrees, minutes, and seconds of arc of a circle, divided by 15, will give the **number** expressing the difference in time between the two places in hours, minutes, and seconds of time. Two cities differing in longitude by $96^\circ 24' 46''$ will differ in time by $(96 \text{ hr. } 24 \text{ min. } 46 \text{ sec.}) \div 15 = 6 \text{ hr. } 25 \text{ min. } 39\frac{1}{5} \text{ sec.}$

If the difference in time between two places is known, their difference in longitude is found by reversing the proc-

ess. Thus, if two cities differ in time by 3 hr. 14 min. 8 sec. they will differ in longitude by

$$(3^\circ 14' 8'') \times 15 = 48^\circ 32' 0''.$$

Never write 4 hr. 14' 35" when you mean 4 hr. 14 min. 35 sec. The 14' and 35" refer to **arcs of a circle**, not to time.

EXERCISES

1. Carry out the experiment with the globe given on page 214. Note about where you live. What cities have noon when you do? What cities have noon earlier than you? later than you?
2. Explain how a difference of 15° in longitude between two places corresponds to a difference of 1 hr. in time; a difference of $15'$ corresponds to a difference of 1 min.; and a difference of $15''$ corresponds to a difference of 1 sec.

Find the difference in longitude between the cities which have the following differences in time:

- | | |
|--------------------------|--------------------------|
| 3. 8 hr. 45 min. 25 sec. | 6. 3 hr. 8 min. 12 sec. |
| 4. 6 hr. 24 min. 52 sec. | 7. 12 hr. 5 min. 34 sec. |
| 5. 2 hr. 37 min. 46 sec. | 8. 5 hr. 56 min. 12 sec. |

From the data on pages 212 and 213 find the difference in time between the following cities:

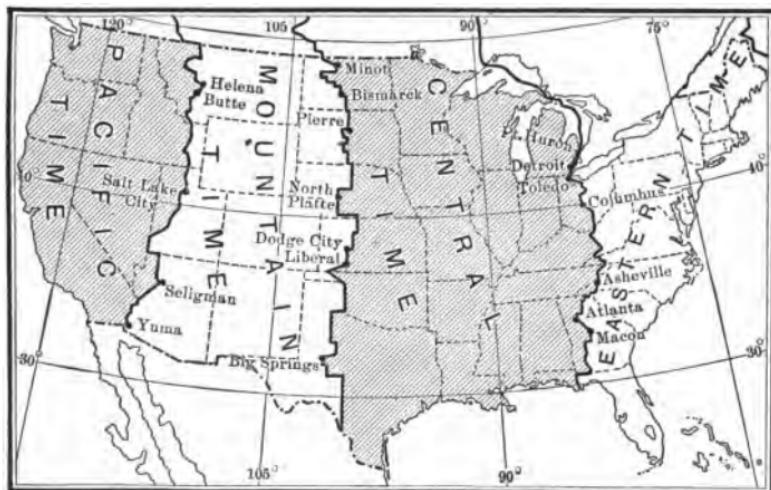
9. New York and Petrograd.
10. New York and Ann Arbor.
11. New York and Paris.
12. Chicago and Paris.
13. Petrograd and Paris.
14. New York and Chicago.
15. Myrtle and Nina have been playing bean bag. Nina's score is + 8, - 5, + 9, - 2, + 10 while Myrtle's score is + 10, - 3, + 5, - 2, + 7. Which girl won?

213. Standard Time.—When the sun is directly south of a place it is noon. Then the shadows fall due north and are shortest. Clocks set to record 12 at this time give local sun time. Suppose that all the clocks in the United States were set to record sun time. A glance at a globe will show that at any instant only clocks upon the same meridian will record the same time; those east will record a later time and those west will record an earlier time. In going east or west a continual change of one's watch would be necessary in order to have correct time. To overcome this difficulty, the railroads in 1883 divided the United States into four time belts. These are shown in the map on the next page. Throughout any belt the sun time of the meridian marked is used. Thus all clocks in any one belt record the same time at any one instant. The clocks in the Eastern Time belt record sun time for meridian $+ 75^\circ$. All clocks in any time belt are exactly one hour faster than the clocks in the time belt next to the west. The clocks in the Central Time belt would then all record sun time of what meridian? The clocks in the Mountain Time belt and in the Pacific Time belt would each record sun time of what meridian?

The lines dividing the time belts are made irregular so as to pass through cities that are railroad division points, because it would be confusing and dangerous to have train men change their time between division points.

Many foreign countries have adopted the use of standard time. Mexico uses the time of the City of Mexico throughout the nation. Western Europe uses the time of longitude 0° ; central Europe uses the time of longitude $- 15^\circ$; eastern Europe uses the time of longitude $- 30^\circ$.

During the summers of 1918 and 1919 the clocks in the United States were moved forward one hour in order to have more usable daylight. Explain.



EXERCISES

Use standard time in solving these exercises.

1. When it is 8 o'clock in Chicago what is the time in Boston? in St. Louis? in Denver? in Peoria? in Seattle?
2. When it is 5 P.M. in Denver what time is it in Chicago? in Los Angeles? in Omaha? in New York City?
3. In what time belt do you live? When your clock is 9.45, what time is it in Topeka, Kan.? in Minneapolis, Minn.? in Portland, Me.? in Portland, Ore.?
4. What change do you make in your watch in crossing the line between any two time belts, if you are travelling east? if you are travelling west?
5. How can the newspapers in San Francisco print at 2 P.M. about any event which happened in Boston at 4 P.M. on the same day?
6. Play a number game, using exponents and radicals.

214. International Date Line.—If you started from Greenwich, England, at noon and could go westward along a parallel at the rate of 15° every hour, you would be continually having noon. When you reach Greenwich again it would still be noon, but the **next day**. When should you have changed from one date to the next? Clearly when you were midway around the globe, or at 180° . The place in the Pacific Ocean where ships change dates is called the **International Date Line**. It does not follow the 180th meridian exactly because it simplifies business to have the line miss land as far as possible, so as not to have two dates in one village or city on the same day. Find a map in a geography or an encyclopedia showing the International Date Line. Use this also in solving the exercises below.

EXERCISES

1. Show how it would be confusing to have the International Date Line pass through the centre of your principal business street. Show why it therefore is made to miss the islands of the Pacific Ocean.
2. When it is 6 A.M. May 3 in San Diego, Cal., what date is it in Chicago, Ill.? in Pekin, China? in Honolulu, Hawaii?
3. What is the difference in longitude between Tokio, Japan, and Los Angeles, Cal.? What is the difference in solar, or sun, time between the two cities?
4. What is the difference in longitude between London, England and Kansas City, Mo.? What is the difference in solar time between these two cities? What is the difference in standard time between these two cities?
5. When it is 5 P.M. May 4 at Denver, Colo., what is the standard time and the date at Hongkong, China? at Petrograd, Russia? at Melbourne, Australia?

215. Multiplication of Signed Numbers.—To multiply the signed number $+ 5ab^2$ by a positive number, as $+ 3$, means to add together 3 of the $+ 5ab^2$. Thus,

$$+ 5ab^2 \times (+ 3) = + 5ab^2 + 5ab^2 + 5ab^2 = 15ab^2.$$

If $+ 5ab^2$ is to be multiplied by $- 3$ then $+ 5ab^2$ is to be subtracted 3 times. Thus,

$$-(+ 5ab^2) - (+ 5ab^2) - (+ 5ab^2) = - 15ab^2.$$

Again, the multiplication of $- 5ab^2$ by $- 3$ is the subtraction of $- 5ab^2$ for 3 times. Thus,

$$- (- 5ab^2) - (- 5ab^2) - (- 5ab^2) = + 15ab^2.$$

Hence, two signed numbers multiplied together give a positive product if their signs are the same; two signed numbers multiplied together give a negative product if their signs are not the same.

216. Plus Omitted.—After this the sign $+$ will be placed before a positive number only when following another number. Thus, $5a^3n - 7g^2h$, $- 73w^2v^3 + 8d^2c$.

EXERCISES

First review the exercises on page 97.

- | | |
|---------------------------|---|
| 1. $3 + 3 + 3 + 3 = ?$ | 9. $- 6 \times (- 7) = ?$ |
| 2. $4 \times 3 = ?$ | 10. $8 \times (- 4ar^2) = ?$ |
| 3. $0 + (- 8) = ?$ | 11. $- 4 \times (- 3er) = ?$ |
| 4. $- 1 \times (8) = ?$ | 12. $4 \times 7mn^2 \times 5 = ?$ |
| 5. $0 - (8) - (8) = ?$ | 13. $5 \times 9r^3 \times 2 = ?$ |
| 6. $- 2 \times (8) = ?$ | 14. $8fg^2 \times (- 3p) = ?$ |
| 7. $0 - (- 8) = ?$ | 15. $- 6w^4r \times (- 8wr^3) = ?$ |
| 8. $- 1 \times (- 8) = ?$ | 16. $- 5 \times 9st^3 \times 4s^2t^7 = ?$ |

17. $4 \times 9cb^3 \times (-5c^2b^7) = ?$
18. $6 \times (-7mn^2) \times (-5m^4n^5) = ?$
19. $-8 \times (-6) \times (-5) = ?$
20. $23 \times (-12) \times (-47) = ?$
21. $9 \times (-5) \times (-2m^2ng^3) = ?$
22. $-8 \times (-4f^2g^5) \times (-5f^6g) = ?$
23. $-5 \times (-6q^3r^2) \times (4qr^5) = ?$
24. $17 \times (-87ac^5) \times (-8a^6c^3) = ?$
25. $-9 \times (-19) \times (-67) \times 24 = ?$
26. $(-5)^2 \times (2)^2 = ?$
27. $(-45)^2 \times (-6)^2 = ?$
28. $(-35)^2 \times (5m^3)^2 = ?$
29. A man lost \$125 on selling a lot. If he lost 4 times as much on a house that he sold, show that this is expressed by $4 \times (-\$125)$. How much is this?
30. Let north distances be positive and south distance be negative. Express 4 times 45 mi. north; 3 times 19 mi. south; 8 times 35 mi. north.
31. Express as signed numbers 5 withdrawals of \$15 each from a bank; 6 withdrawals of \$25 each; 7 withdrawals of \$30 each. Find the amount of each withdrawal and the sum.
32. Express as signed numbers buying 5 cords of wood at \$3.50 per cord; selling 96 bu. wheat at \$1.15 per bushel.
33. A bar of aluminum expands an amount equal to its length times .000023, times the degrees its temperature has increased on the Centigrade scale. Express the increase in a bar of aluminum 5.5 ft. long whose temperature has increased 100° ; decreased 50° ; increased 45° ; decreased 65° .
- 217. Division of Signed Numbers.**—Division of signed numbers is the reverse of multiplication just as it is the reverse of multiplication in every kind of numbers. In

finding $10 \div (-2)$ the number is found which multiplied by -2 gives 10 . This number is -5 . Why is that so? In dividing one literal number by another it has previously been necessary to find: (1) the numerical coefficient; (2) the literal numbers present; (3) the exponent of each literal number. In addition we must now find the sign when we are dealing with signed numbers. Thus, $(-20a^3b^2) \div (4ab) = -5a^2b$. The sign is $-$ because we must multiply the positive $4ab$ by a negative number to get $-20a^3b^2$. $20 \div 4 = ?$ $a^3 \div a = ?$ $b^2 \div b = ?$ Division of signed literal numbers will be simple if division is thought of as the reverse, or the opposite, of multiplication.

EXERCISES

First work all the exercises on page 98.

Carry out the following operations:

- | | |
|--|-------------------------------------|
| 1. -4×9 | 9. $42g^5R^3 \div 21g^3R^3$ |
| 2. $-8 \div 4$ | 10. $-72J^7H^3 \div (-12J^3)$ |
| 3. $-125 \div 25$ | 11. $-48TR^3 \div 6TR^2$ |
| 4. $-25 \div (-25)$ | 12. $67K^7H^3R^2 \div (-23K^3H^2R)$ |
| 5. $-35 \div (-7)$ | 13. $-48m^3y^3 \div 8m^2y^2$ |
| 6. $-89 \times (-76)$ | 14. $56S^2R^3 \div (-7R^2)$ |
| 7. $-32qr^3 \div 4qr$ | 15. $-81a^3x^5 \div (-27ax^3)$ |
| 8. $-49m^3h^2 \div 7m^3h$ | 16. $-6ar \times (-9ar^2)$ |
| 17. $-4m^3h^3 \times (7m^3k^2) \div 14mhk$ | |
| 18. $-8a^3 \times (-5a^2) \div 20a$ | |
| 19. $7a^2d^3 \times 6ad^2 \div 21a^3d^3$ | |
| 20. $18ft^5 \times (-7t^3) \times (-5)$ | |
| 21. $24ft^3 \div (-16t^2) \times 4f^3$ | |
| 22. $-60w^6N^5 \div (-12w^3N^2)$ | |
| 23. $42a^7 \times (-5a^3) \div (-30a^5)$ | |
| 24. $63grt^3 \div (-7grt^2)$ | |

218. Equations with Negative Roots.—Equations may have negative roots just as well as positive roots, although so far we have found only positive roots. Suppose that V in the equation at the bottom of this page was less than 332, say 320, then,

$$320 = 332 + 0.6t \quad (1)$$

$$320 - 332 = 0.6t \quad (2)$$

$$-12 = 0.6t \quad (3)$$

and $-12 \div 0.6 = t$ (4)

hence, $-20 = t$, (5)

or the thermometer is 20° below zero.

Check this value of the root for (1) by substituting its value in that equation.

EXERCISES

Find the roots for the following equations and check the results:

- | | |
|-------------------------|-------------------------|
| 1. $5m - 7 = 3m + 5$ | 13. $5E - 6 = 12 - E$ |
| 2. $7w + 9 = 2w + 34$ | 14. $2p + 7 = 19 - 4p$ |
| 3. $8L + 7 = 5L - 11$ | 15. $8 - 7j = 5j + 32$ |
| 4. $7B + 5 = 4B - 10$ | 16. $2S - 5 = 6S - 9$ |
| 5. $13g - 7 = 8g - 32$ | 17. $7Z + 9 = 11Z - 7$ |
| 6. $17K + 9 = -25$ | 18. $4D - 5 = 7 - 2D$ |
| 7. $5q + 17 = 33 - 3q$ | 19. $13 - 2K = 5K - 8$ |
| 8. $7 + 3G = 9 + G$ | 20. $15 - 4F = 2F - 3$ |
| 9. $5C - 12 = 2C + 3$ | 21. $7U + 17 = 3U - 15$ |
| 10. $4R + 5 = -39$ | 22. $19M + 42 = 12M$ |
| 11. $13 - 2N = 3N - 7$ | 23. $27 - 7Y = -2Y - 3$ |
| 12. $8Y - 13 = 20 - 3Y$ | 24. $8T - 16 = 4T - 14$ |

The equation of the velocity, in meters, of sound through air is

$$V = 332 + 0.6t. \quad (6)$$

t is the reading of the Centigrade scale in degrees. The $0.6t$ is the increase in meters for the increase in temperature of t degrees.

25. What is the velocity of sound at zero degrees? How much more is it at $+ 10^\circ$? What is the velocity at $+ 10^\circ$? What is the velocity at $- 10^\circ$?
26. How much less is the velocity of sound at $- 20^\circ$ than it is at zero degrees? What is the velocity at $- 20^\circ$?
27. How much less is the velocity of sound at $- 30^\circ$ than it is at $+ 20^\circ$? less at $- 14^\circ$ than it is at $+ 26^\circ$? Solve this problem without first finding the velocity at the two temperatures given.
28. Use equation (6) to find the temperature, t , at which the velocity, V , will be 324 meters per second.
29. Use equation (6) to find the temperature, t , at which the velocity, V , will be 328 meters per second.
30. Travelling salesmen go back and forth so as to "make" as many towns in a day as possible. One day a salesman made the following trips: north 35 mi., south 16 mi., north 42 mi., south 23 mi. Express each as a signed number and find their sum. Also interpret this result.
31. If we write $+ 1919$ for 1919 A.D., how should we write 287 B.C., the year in which the great mathematician Archimedes was born? Set up the expression telling how long ago Archimedes was born.
32. Pythagoras who discovered the Pythagorean theorem was born in 580 B.C. State this as a signed number. State as a signed number the time from his birth to the present year. State as a signed number the time from this year to 580 B.C.

33. Another great mathematician, Boethius, was born in 480 A.D. How long ago was that? How long after the birth of Archimedes was Boethius born?

34. Show by the use of signed numbers that the following equation connecting total sales, expenses, and profits or losses, is a true equation:

$$S - E = P, \text{ or } L.$$

Find the profit or loss by the use of the above equation when sales are \$ 436 and expenses are \$ 259.

35. Use the above equation to find the profit or loss when the total sales are \$ 675 and the expenses are \$ 857.

36. By the use of the above equation find the profit or loss when total sales are \$ 1325 and the total expenses are \$ 1148.

By the use of signed numbers the actual velocity, V , that a man rows in a stream, either with or against the current, is given by the equation,

$$V = V_m + V_s.$$

V_m and V_s are the velocities of the man rowing in still water and of the stream.

37. What would be the meaning of a negative number for V ? Explain the equation fully.

38. A man can row 4 mi. per hour in still water. How fast can he row down a stream moving at the rate of 1.5 mi. per hour?

39. A man can row 3.5 mi. per hour in still water. How fast can he row against a current moving 2 mi. per hour?

40. Harry can row 2 mi. per hour in still water. Show by the above equation what his motion will be rowing down a stream moving at the rate of 3 mi. per hour.

41. Show by means of the above equation what Harry's motion would be rowing against the current.

REFERENCE TABLES

ENGLISH SYSTEM

Length

12 inches (in.) = 1 foot (ft.).

3 feet = 1 yard (yd.).

5½ yards, or 16½ feet = 1 rod (rd.).

320 rods, or 5280 feet = 1 mile (mi.).

5 ft. 3 in. may be written 5' 3".

SURVEYORS' TABLE OF LENGTH

7.92 inches = 1 link (li.).

100 links = 4 rods = 1 chain (ch.).

80 chains = 5280 ft. = 1 mile.

ADDITIONAL UNITS OF LENGTH

4 inches = 1 hand.

6 feet = 1 fathom.

100 fathoms = 1 cable length.

1.15 common miles = 1 knot (nautical mile).

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).

9 square feet = 1 square yard (sq. yd.).

30½ square yards = 1 square rod (sq. rd.).

160 square rods = 1 acre (A.).

640 acres = 1 square mile (sq. mi.).

SURVEYORS' TABLE OF SQUARE MEASURE

16 square rods (sq. rd.) = 1 square chain (sq. ch.).

10 square chains = 1 acre (A.).

640 acres = 1 square mile (sq. mi.).

36 square miles = 1 township (tp.).

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
 27 cubic feet = 1 cubic yard (cu. yd.).
 128 cubic feet = 1 cord (cd.).
 24 $\frac{1}{2}$ cubic feet = 1 perch (stone, etc.).
 A cubic yard is called a load.

Weight

AVOIRDUPOIS

16 ounces (oz.) = 1 pound (lb.).
 100 pounds = 1 hundredweight (cwt.).
 2000 pounds = 1 ton (T.).
 112 pounds = 1 long hundredweight.
 2240 pounds = 1 long ton.

Long ton is used in U. S. Custom-house and at mines.
 1 ton soft coal occupies about 35 cubic feet.
 1 ton hard coal occupies about 28 cubic feet.

Liquid Measure

4 gills (gi.) = 1 pint (pt.).
 2 pints = 1 quart (qt.).
 4 quarts = 1 gallon (gal.).
 231 cubic inches = 1 gallon.
 31.5 gallons = 1 barrel (bbl.) (varies).
 2 barrels = 1 hogshead (varies).
 1 pint = 16 fluid ounces (apothecaries').
 57.75 cubic inches = 1 liquid quart.
 $\frac{1}{2}$ pint = 1 measuring cup.
 1 cubic foot of water weighs nearly $62\frac{1}{2}$ pounds.
 1 gallon of water weighs nearly $8\frac{1}{2}$ pounds.
 1 cubic foot of water equals about $7\frac{1}{2}$ gallons.

Dry Measure

2 pints (pt.) = 1 quart (qt.).
 8 quarts = 1 peck (pk.).
 4 pecks = 1 bushel (bu.).
 2150.4 cubic inches = 1 bushel.
 1 stricken bushel = $1\frac{1}{2}$ cubic feet (nearly).
 1 heaped bushel = $1\frac{1}{2}$ cubic feet (nearly).
 1 bushel ear corn = $2\frac{1}{2}$ cubic feet (nearly).

THE FOLLOWING HOLD IN NEARLY ALL STATES

- 1 bushel of wheat weighs 60 pounds.
 1 bushel of shelled corn weighs 56 pounds.
 1 bushel of ear corn weighs 75 or 80 pounds in the fall and 70 pounds later.
 1 bushel of oats weighs 32 pounds.
 1 bushel of rye weighs 56 pounds.
 1 bushel of barley weighs 48 pounds.
 1 bushel of potatoes weighs 60 pounds.
 1 bushel of beans weighs 60 pounds.
 1 bushel of peas weighs 60 pounds.
 1 bushel of apples weighs 48 pounds.
 1 bushel of clover seed weighs 60 pounds.
 1 bushel of alfalfa seed weighs 60 pounds.
 1 bushel of timothy seed weighs 45 pounds.
 1 bushel of bran weighs 20 pounds.
 1 bushel of soft coal weighs 80 pounds.
 1 barrel of flour weighs 196 pounds.
 1 barrel of pork or beef weighs 200 pounds.

For hay measurements in the stack, see page 190.

Angles and Arcs

- 60 seconds ($60''$) = 1 minute ($1'$).
 60 minutes = 1 degree (1°).
 90 degrees = 1 right angle.
 360 degrees = 1 circumference.

Time

- 60 seconds (sec.) = 1 minute (min.).
 60 minutes = 1 hour (hr.).
 24 hours = 1 day (da.).
 7 days = 1 week (wk.).
 12 months (mo.) = 1 year (yr.).
 365 days = 1 common year.
 366 days = 1 leap year.

United States Money

- 10 mills = 1 cent (ct. or ¢).
 10 cents = 1 dime (d.).
 10 dimes = 1 dollar (\$).
 10 dollars = 1 eagle (E.).

Counting

12 units = 1 dozen (doz.).

12 dozen, or 144 = 1 gross (gr.).

12 gross, or 1728 = 1 great gross.

20 units = 1 score.

The dozen is being replaced by 10 and the gross by 100.

500 sheets of paper are called a ream.

METRIC SYSTEM

Length

10 millimeters (mm.) = 1 centimeter (cm.).

10 centimeters = 1 decimeter (dm.).

10 decimeters = 1 meter (m.).

10 meters = 1 Dekameter (Dm.).

10 Dekameters = 1 Hektometer (Hm.).

10 Hektometers = 1 Kilometer (Km.).

Square Measure

100 square millimeters (mm^2) = 1 square centimeter (cm^2).

100 square centimeters (cm^2) = 1 square decimeter (dm^2).

100 square decimeters = 1 square meter (m^2).

100 square meters = 1 square Dekameter (Dm^2).

100 square Dekameters = 1 square Hektometer (Hm^2).

100 square Hektometers = 1 square Kilometer (Km^2).

A square Dekameter is called an **are**. As 100 square Dekameters equals 1 square Hektometer, a square Hektometer is called a **hektare** (ha.). These are the metric units of land measure.

Cubic Measure

1000 cubic millimeters (mm^3) = 1 cubic centimeter (cm^3).

1000 cubic centimeters = 1 cubic decimeter (dm^3).

1000 cubic decimeters = 1 cubic meter (m^3).

1000 cubic meters = 1 cubic Dekameter (Dm^3).

1000 cubic Dekameters = 1 cubic Hektometer (Hm^3).

1000 cubic Hektometers = 1 cubic Kilometer (Km^3).

The cubic meter is used in measuring wood and is called the **stere** (st.).

Weight

- 10 milligrams (mg.) = 1 centigram (cg.).
 10 centigrams = 1 decigram (dg.).
 10 decigrams = 1 gram (g.).
 10 grams = 1 Dekagram (Dg.).
 10 Dekagrams = 1 Hektogram (Hg.).
 10 Hektograms = 1 Kilogram (Kg.).
 1000 Kilograms = 1 Metric ton (t.).

The gram is the weight of 1 cm³. of water at a temperature of 4° C.

Capacity

- 10 milliliters (ml.) = 1 centiliter (cl.).
 10 centiliters = 1 deciliter (dl.).
 10 deciliters = 1 liter (l.).
 10 liters = 1 Dekaliter (Dl.).
 10 Dekaliters = 1 Hektoliter (Hl.).
 10 Hektoliters = 1 Kiloliter (Kl.).

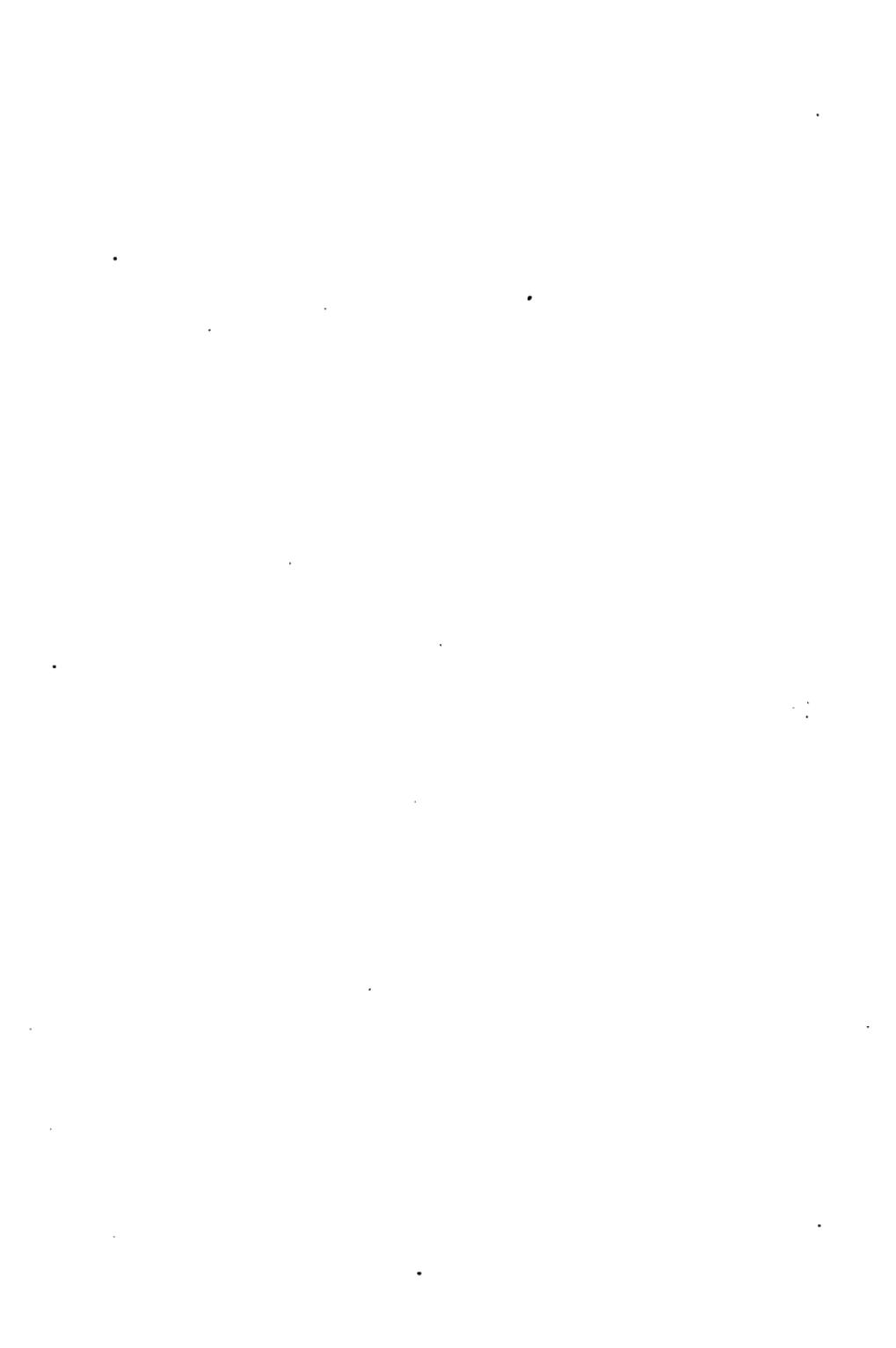
The liter is 1 dm³.

Equivalents

A meter	= 39.37 in.	= 3½ ft. (nearly).
A Kilometer	= .621 mi.	= .6 mi. (nearly).
A liter	= 1.056 qt. (liquid)	= 1 qt. (nearly).
A liter	= .908 qt. (dry)	= .9 qt. (nearly).
A Kilogram	= 2.204 lb.	= 2.2 lb. (nearly).
A hectare	= 2.47 A.	= 2½ A. (nearly).

Formulas of Areas and Volumes

Area parallelogram	= ab.
Area triangle	= ½ ab.
Area trapezoid	= ½ a(B + b).
Area circle	= π R ² .
Area ring	= π (R ² - r ²).
Circumference circle	= 2 π R.
Lateral surface regular pyramid	= ½ nls.
Lateral surface cone	= π Rs.
Surface sphere	= 4 π R ² .
Volume pyramid	= ½ Bh.
Volume cone	= ½ π R ² h.
Volume sphere	= ½ π R ³ .



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